Computational Challenges for Subsurface Flow Modeling and Optimization

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Computational Issues for Oil / Gas Production and CO_2 Sequestration

- Multiscale geology
- Unconventional resources and complex recovery processes increasingly important
  - Tar sands, oil shales, shale gas, coalbed methane
  - Recovery / CO_2 storage: multiphysics & multiscale
- Use of computational optimization and data assimilation procedures; multiple objective optimization
- Uncertainty assessment, risk management

Darcy Flow through Porous Formations

- Darcy velocity ***u***, volumetric flow rate ***q*** ($u = q/A$)

$p = p_1$  $p = p_2$

$\longleftarrow L \longrightarrow$

rock permeability k, fluid viscosity μ, porosity ϕ

$$u = -\frac{k}{\mu}\frac{dp}{dx}, \quad \phi = \frac{V_{\text{pore}}}{V_{\text{bulk}}}$$

- In multiple dimensions, $\mathbf{u} = -\frac{1}{\mu}\mathbf{k}\nabla p$

Governing Equations for Subsurface Flow
(Multicomponent, Multiphase, Darcy Scale)

- Component mass balances for ***component i***:

$$\frac{\partial}{\partial t}\left(\sum_j \phi\rho_j y_{ij} S_j\right) + \nabla\cdot\left(\sum_j \rho_j y_{ij}\mathbf{u}_j\right) = m_c \qquad \boldsymbol{n_c}\ \textbf{equations}$$

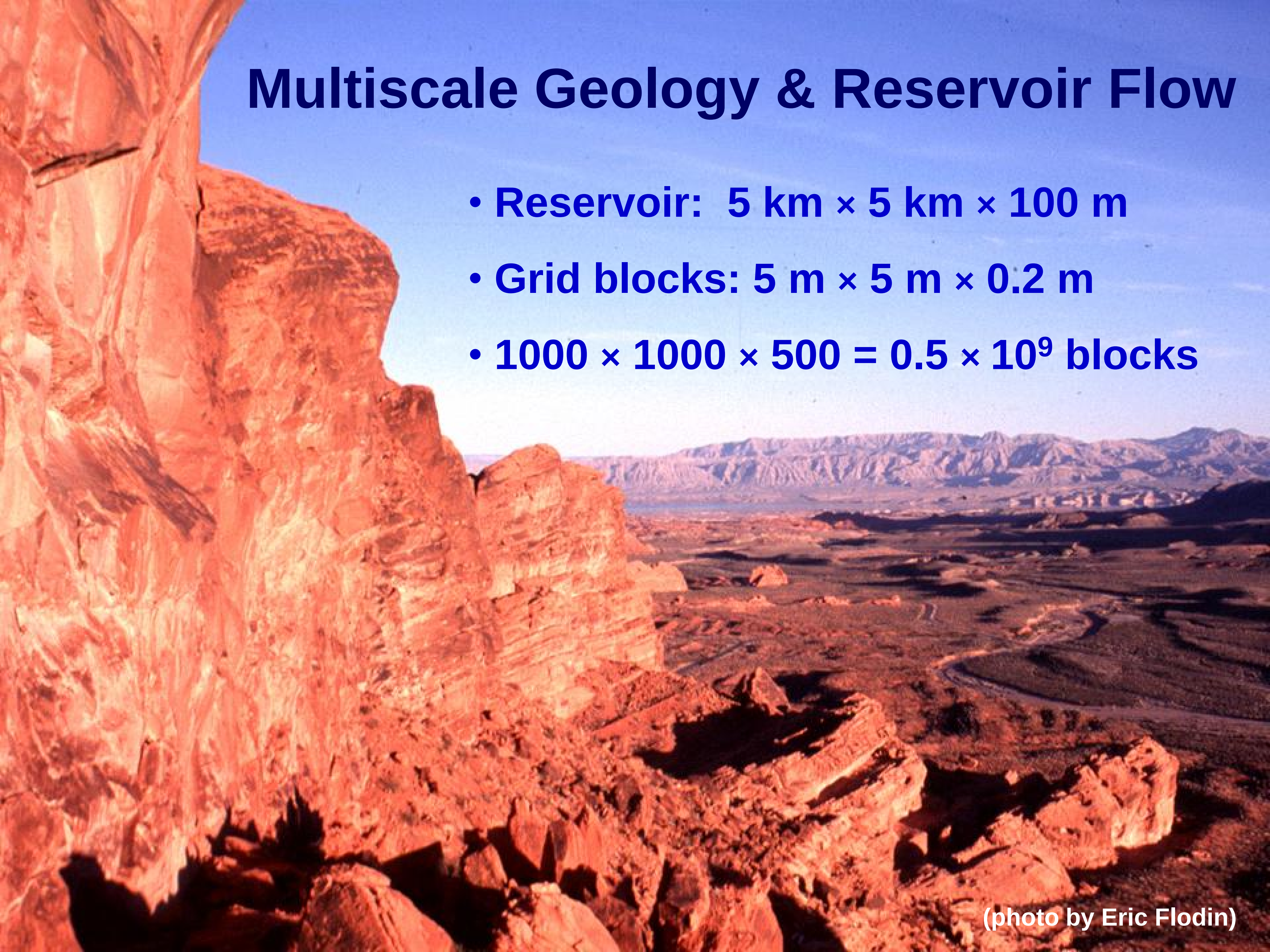
- Darcy's laws for ***phase j*** : $\mathbf{u}_j = -\frac{k_{rj}(S)}{\mu_{\mathrm{j}}}\mathbf{k}\nabla p_j$

- Equilibrium relationships: $K = y_{i,vapor} / y_{i,oleic}$

- Additional (problem specific) effects: energy equation, geomechanics, chemical reactions, …

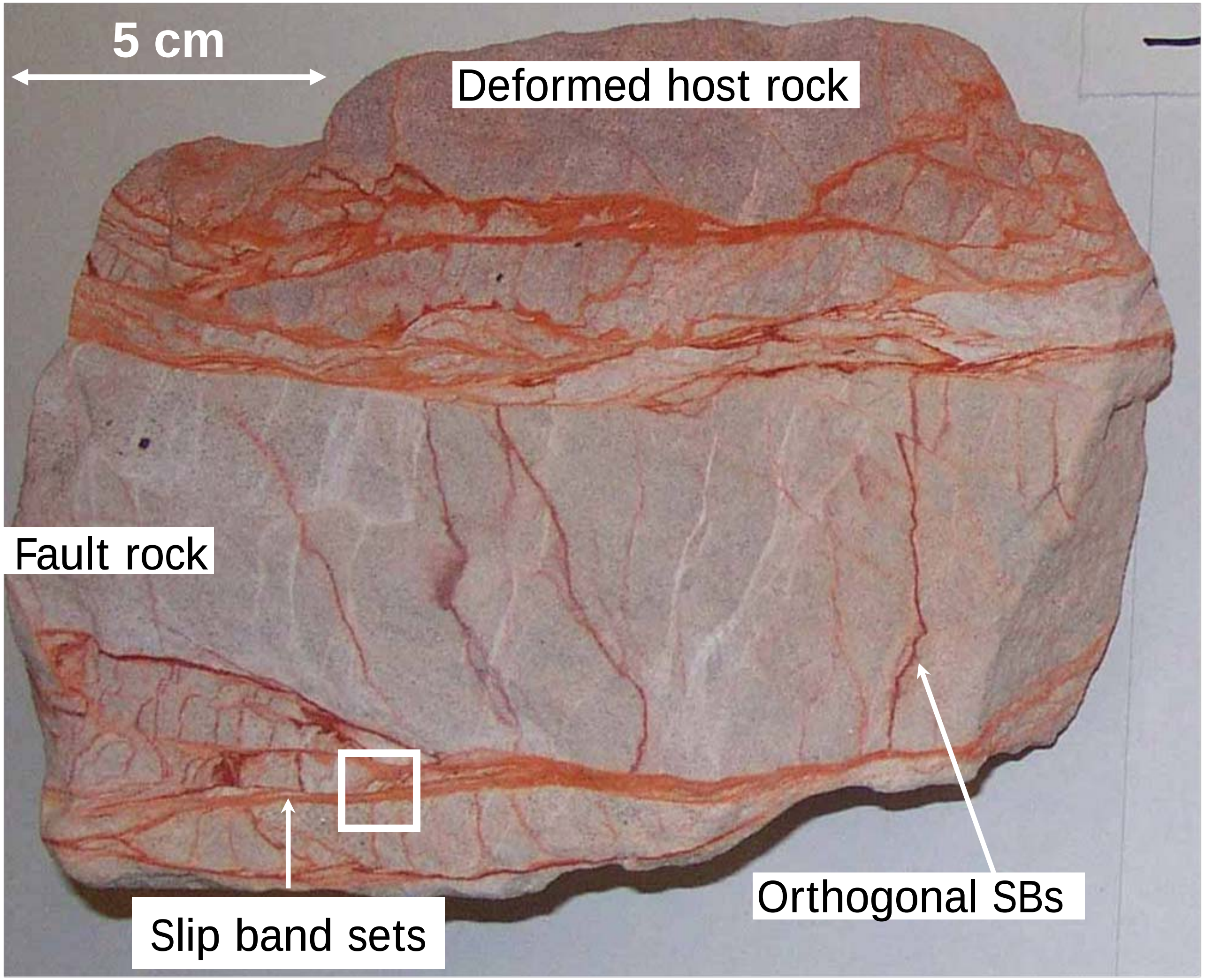
Multiscale Geology & Reservoir Flow

- Reservoir: 5 km × 5 km × 100 m
- Grid blocks: 5 m × 5 m × 0.2 m
- $1000 \times 1000 \times 500 = 0.5 \times 10^9$ blocks



(photo by Eric Flodin)

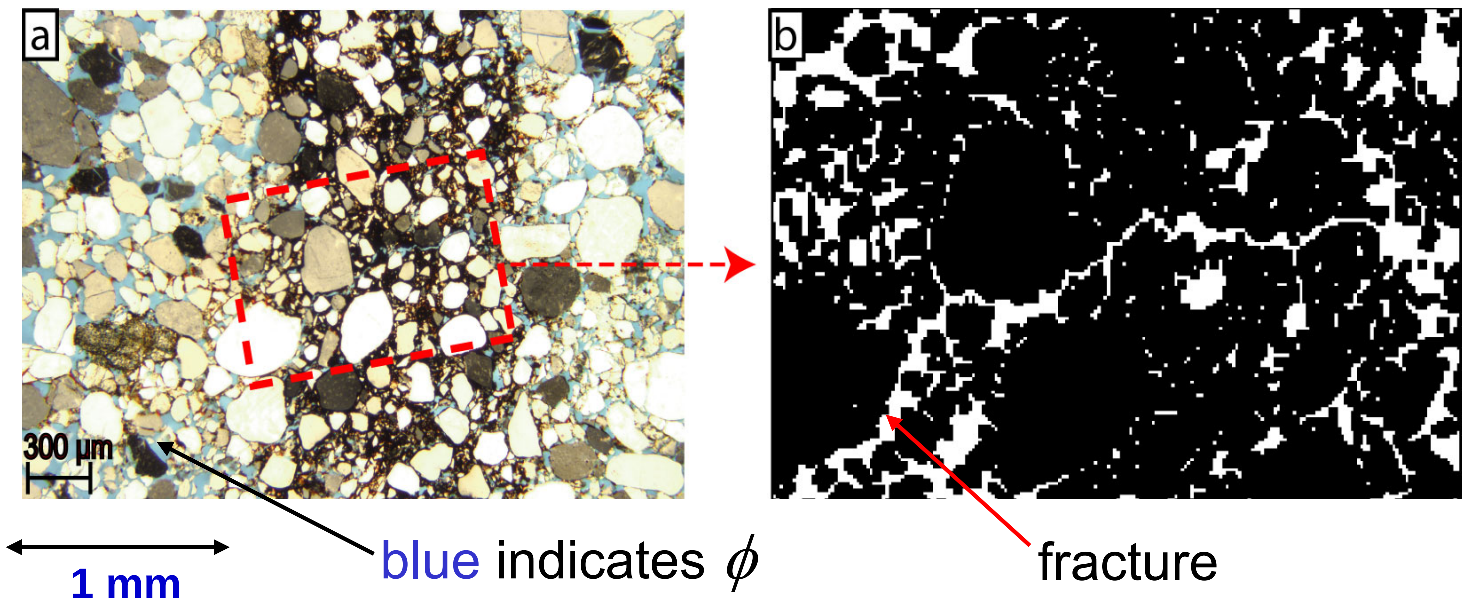
Fault Zone Features



(from Ahmadov et al., 2007)

Thin Section of Slip Bands

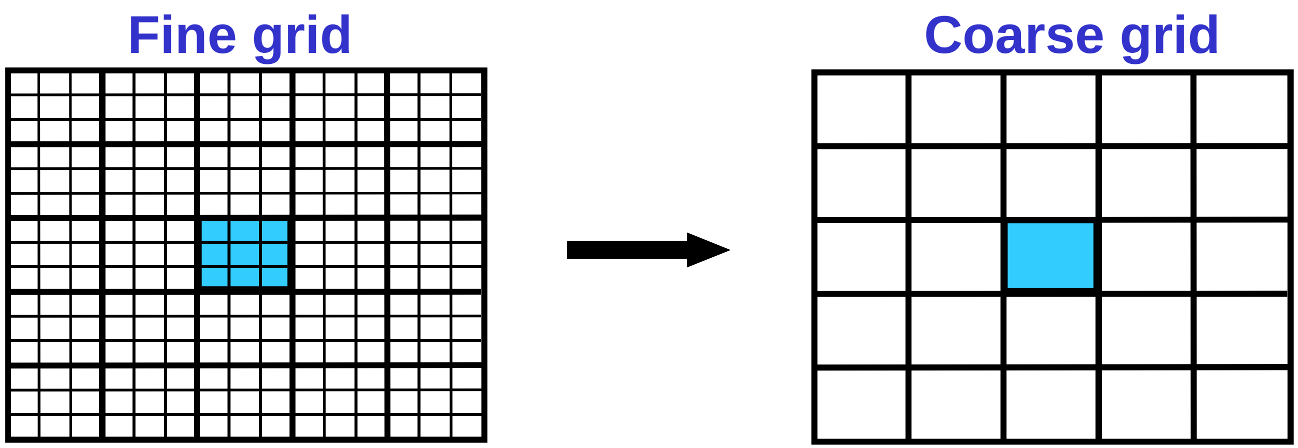
- Low ϕ and k in slip bands; presence of open fractures may have strong impact on flow



(from Ahmadov et al., 2007)

Upscaling Approaches

- Upscaling (computational homogenization) methods provide averaged or coarse-grained results



$$\nabla \cdot \left(\lambda(S)\mathbf{k} \cdot \nabla p \right) = \tilde{q}_t \qquad \nabla \cdot \left(\lambda^*(S^c)\mathbf{k}^* \cdot \nabla p^c \right) = \tilde{q}_t$$

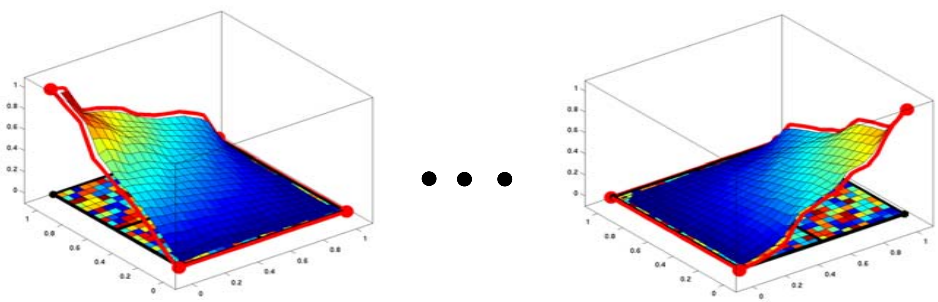
$$\phi \frac{\partial S}{\partial t} + \nabla \cdot \left(\mathbf{u} f(S) \right) = -\tilde{q}_w \qquad \phi^* \frac{\partial S^c}{\partial t} + \nabla \cdot \left(\mathbf{u}^c f^*(S^c) \right) = -\tilde{q}_w$$

coarse functions ($\mathbf{k}^*$, ϕ^*, λ^*, f^*) generally precomputed

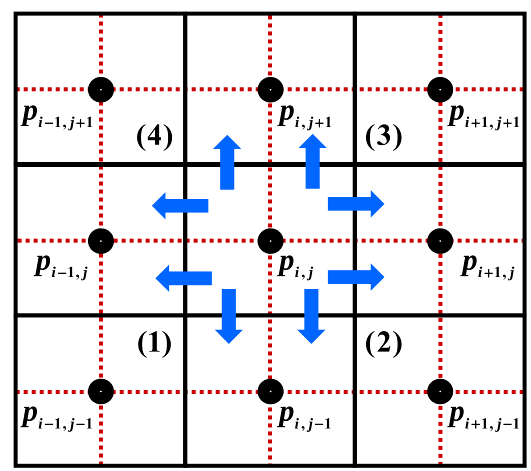
Multiscale Finite Element / Volume Methods

- Construct (local) basis functions to capture subgrid **k(x)**
- Reconstruct fine-scale **u** for transport calculations
- **FE** (Hou, Wu, Efendiev, …), **FV** (Jenny, Lee, Tchelepi, Lunati, …), **MFE** (Arbogast, Aarnes, Krogstad, Lie, Juanes, Chen …)

MsFVM



basis functions



primal & dual grids

(figures from Hesse, Mallison, Tchelepi)

Reduced-Order Modeling Procedures

- ROM approaches require "training" run using full-order model [$\mathbf{g}(\mathbf{x})=0$] and basis construction

- "Snapshot" matrix $\mathbf{X}=[\mathbf{x}_1, \mathbf{x}_2, \ldots \mathbf{x}_N]$; SVD of $\mathbf{X}$ provides basis matrix $\mathbf{\Phi}$ ($\mathbf{x}=\mathbf{\Phi}\mathbf{z}$)

- Basic POD: $\mathbf{J}\boldsymbol{\delta} = -\mathbf{g} \;\rightarrow\; \left(\mathbf{\Phi}^T\mathbf{J}\mathbf{\Phi}\right)\boldsymbol{\delta}_r = -\mathbf{\Phi}^T\mathbf{g}$

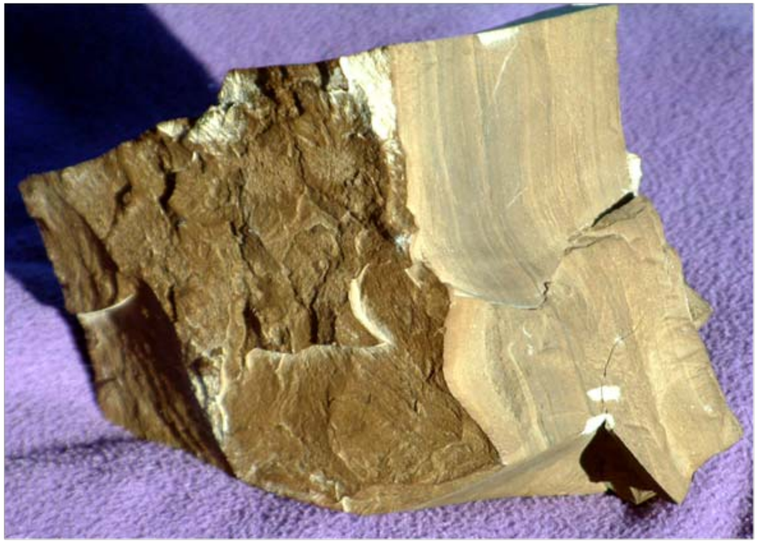
- Trajectory piecewise linearization (TPWL):

$$\mathbf{z}^{n+1} = \mathbf{z}^{i+1} - \left(\mathbf{J}_r^{i+1}\right)^{-1}\left[\left(\frac{\partial \mathbf{g}^{i+1}}{\partial \mathbf{x}^i}\right)_r (\mathbf{z}^n - \mathbf{z}^i) + \left(\frac{\partial \mathbf{g}^{i+1}}{\partial \mathbf{u}^{i+1}}\right)_r (\mathbf{u}^{n+1} - \mathbf{u}^{i+1})\right]$$

$$\mathbf{J}_r^{i+1} = \mathbf{\Phi}^T\mathbf{J}^{i+1}\mathbf{\Phi}$$

Oil from Unconventional Sources

tar sands

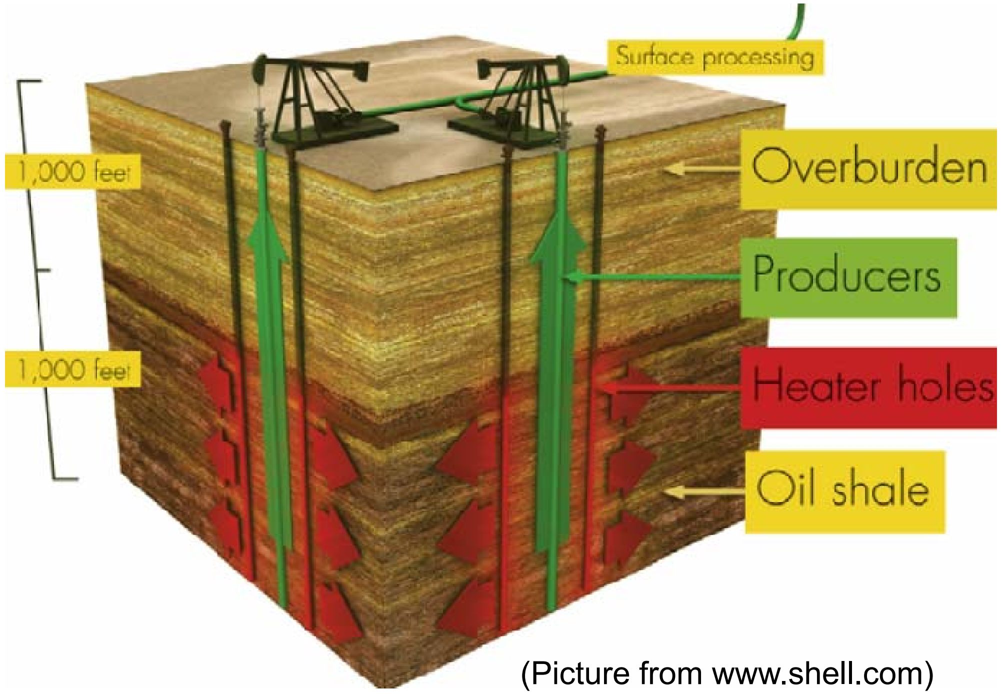


oil shales

- Alberta tar sands: ~170 × 10^9 bbl in ***reserves***
- US oil shales: ~2 × 10^{12} bbl ***resource*** (Green River Formation, UT, CO, WY)

pictures from http://www.technologyreview.com/NanoTech/wtr_16059,318,p1.html (left), S. Graham (right)

In-situ Upgrading of Oil Shale

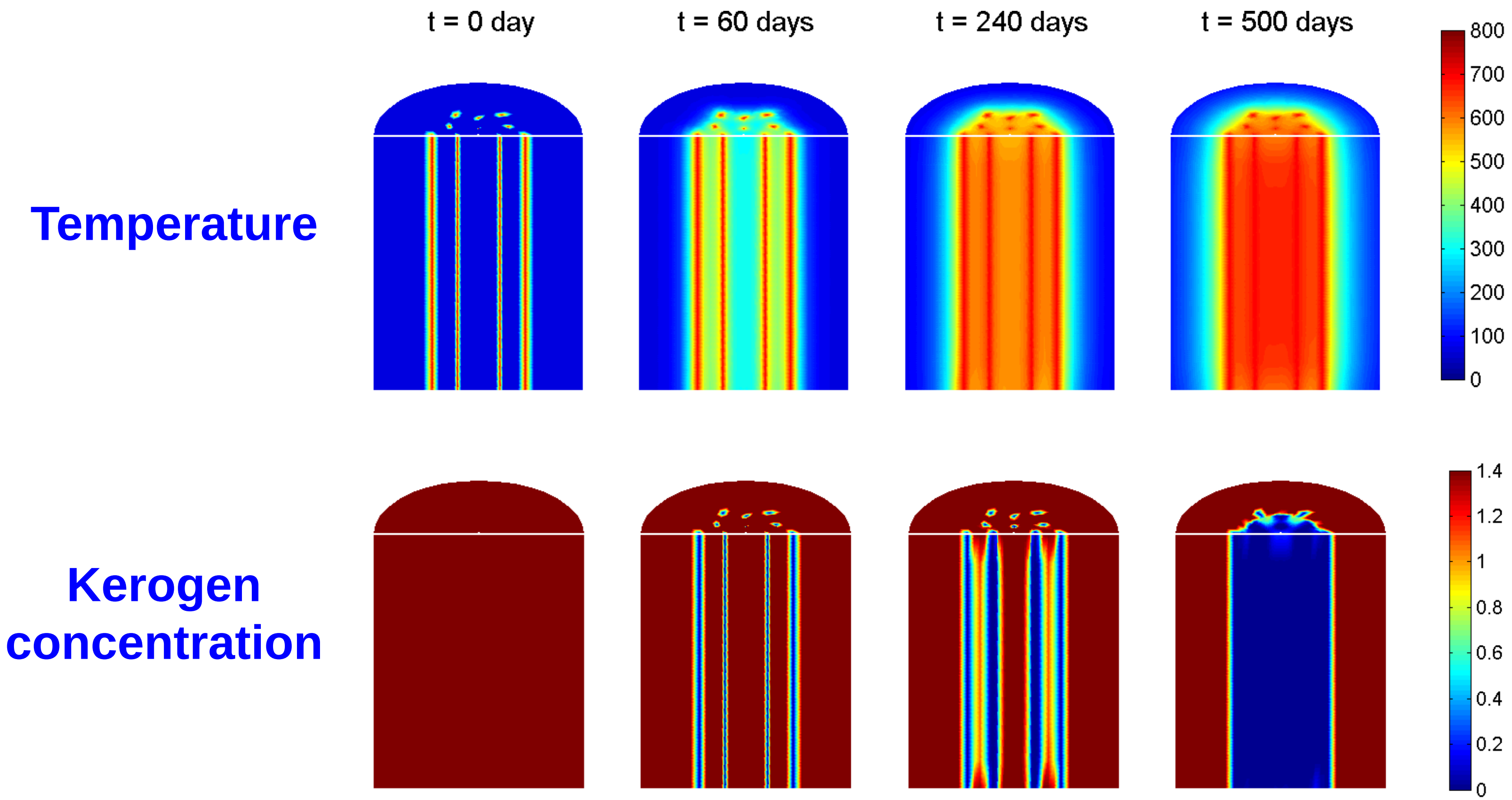


(Picture from www.shell.com)

kerogen(s) $\rightarrow$ oil + gas

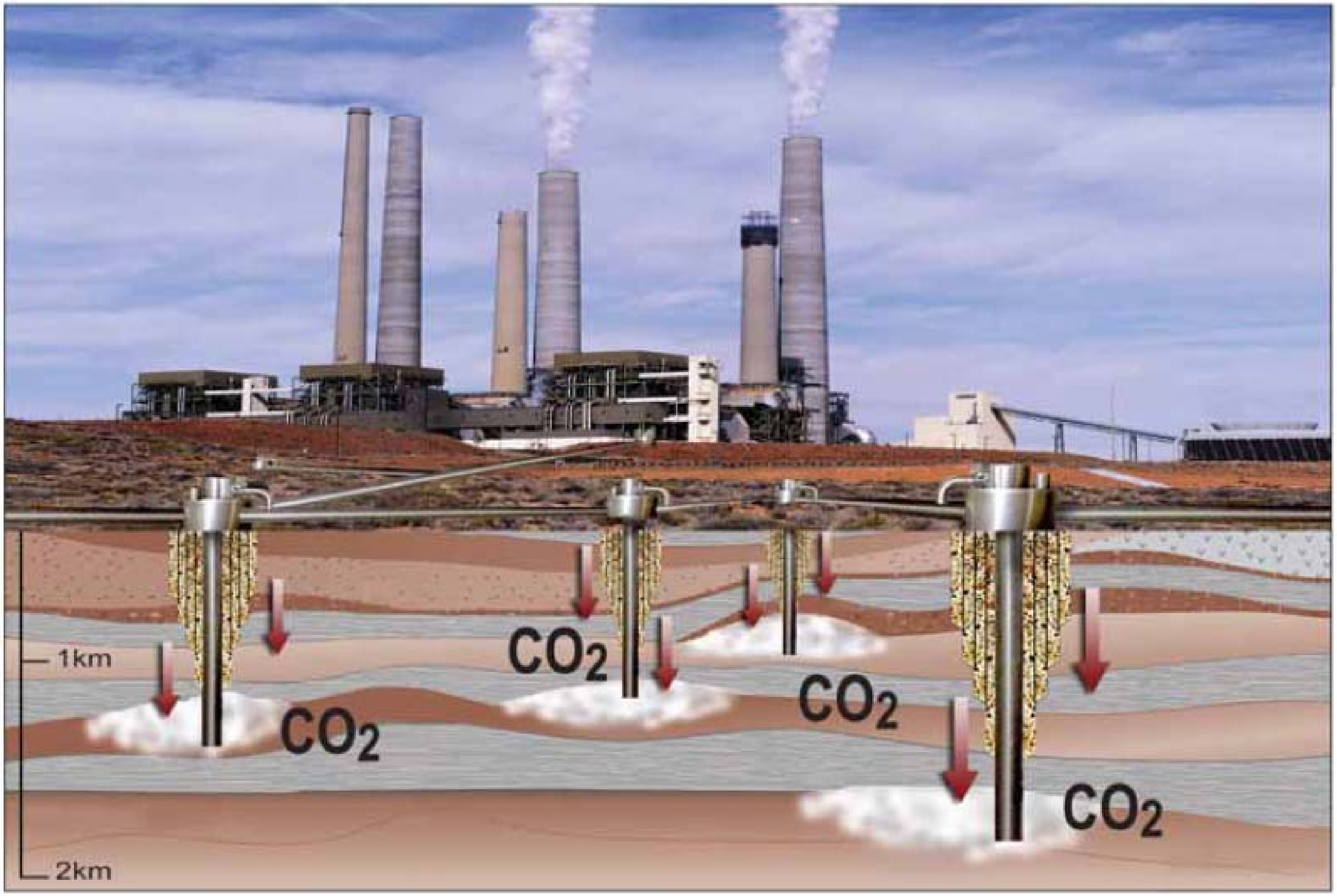
(via several reactions)

In-situ Upgrading Simulations



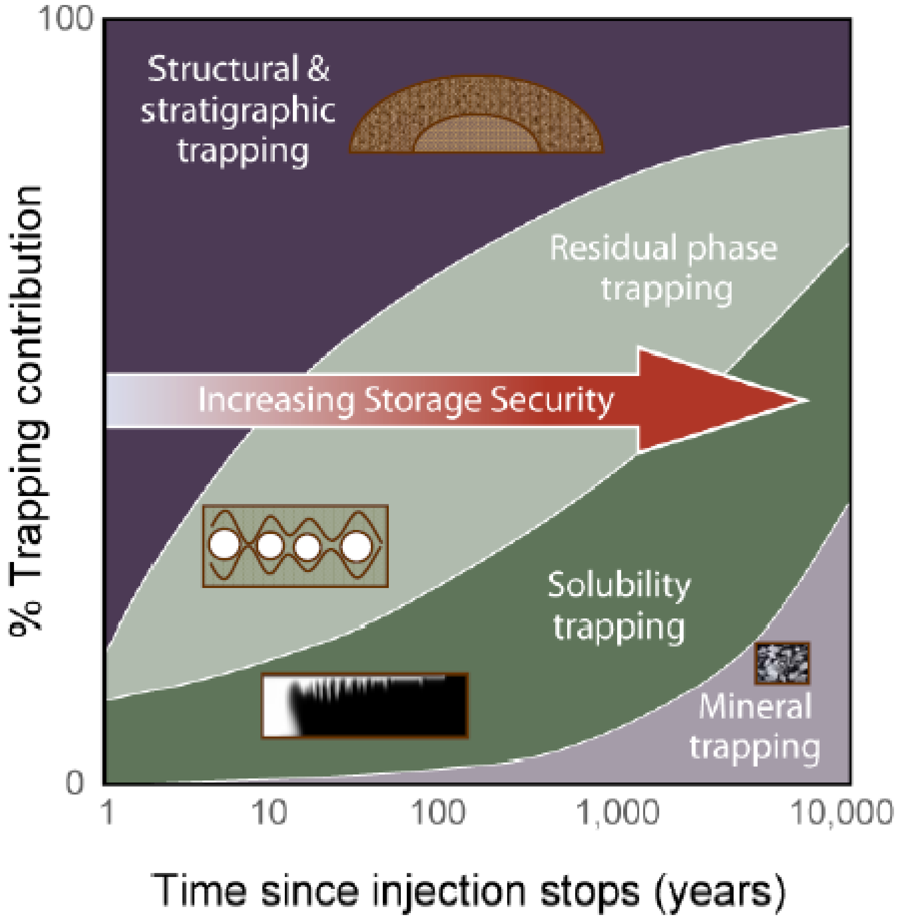
Fan, Durlofsky & Tchelepi (2009)

Geological Sequestration of CO_2



(from Benson, 2007)

Geologic Storage of Carbon



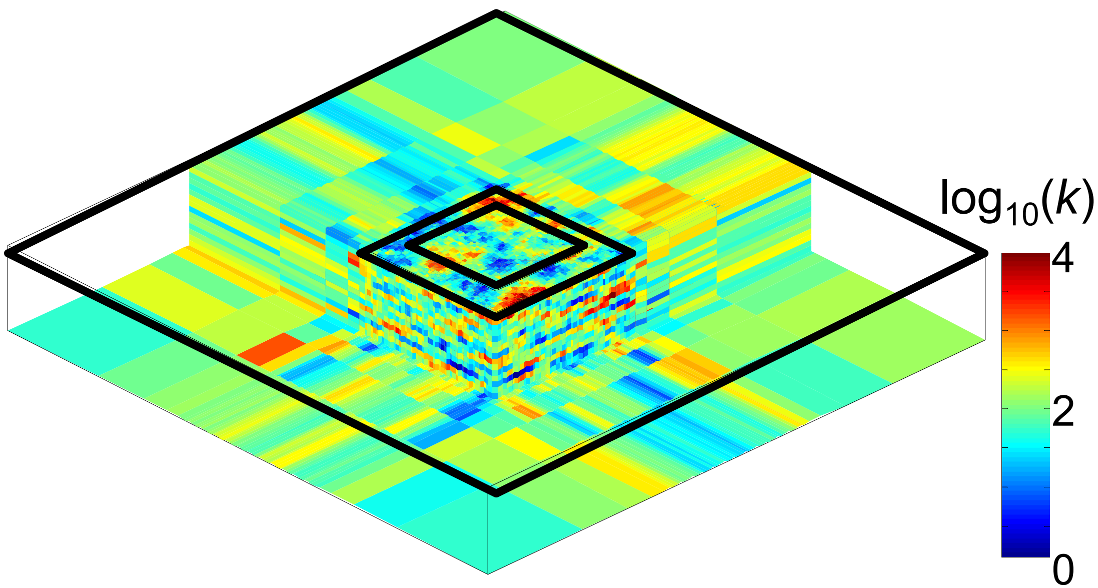
<u>**Trapping Mechanisms**</u>

- **Structural**
- **Residual**
- **Solubility**
- **Mineral**

(from Benson, 2007)

Optimization of Well Placement and Settings

- Determine well locations, orientation and injection schedule to minimize mobile CO_2
- Applying global stochastic search (PSO) and local search (GPS, HJ) optimizers



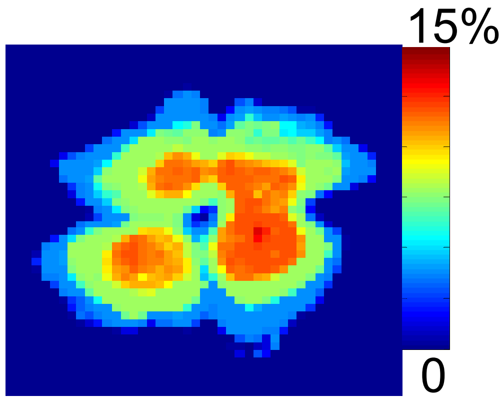
(work with David Cameron)

Optimization Results

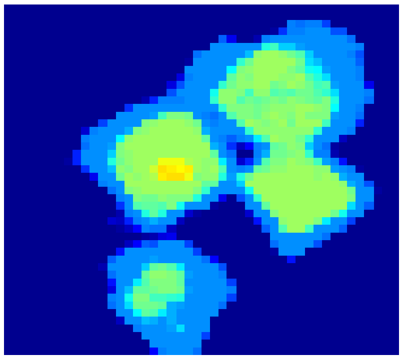
Optimal Injection Strategy (4 periods)

Mobile CO_2

(top view, vertically averaged, low hysteresis)



Default



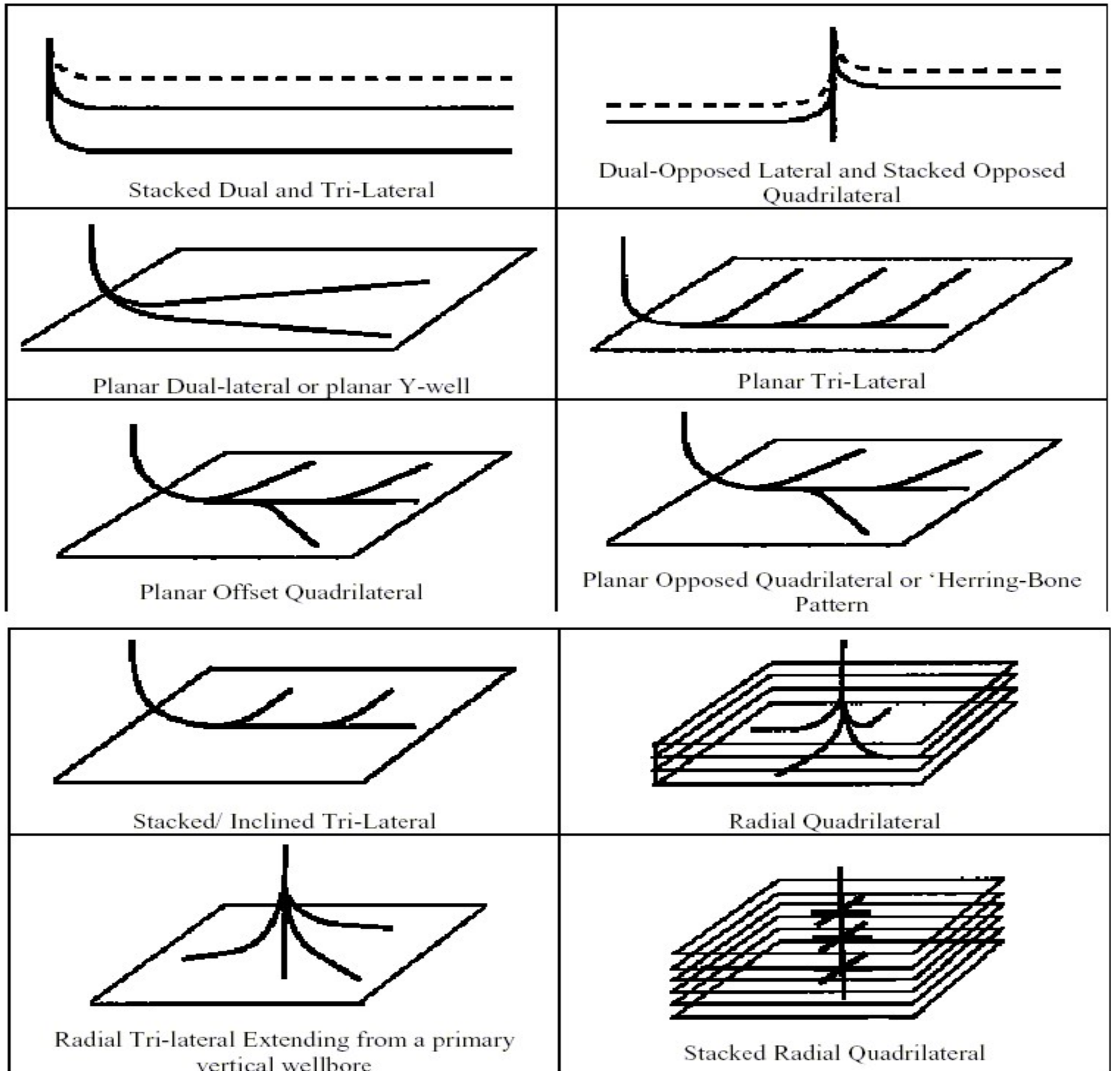
Optimized

Optimization of Field Development

- Goal is, e.g., to maximize net present value of multi-well development project

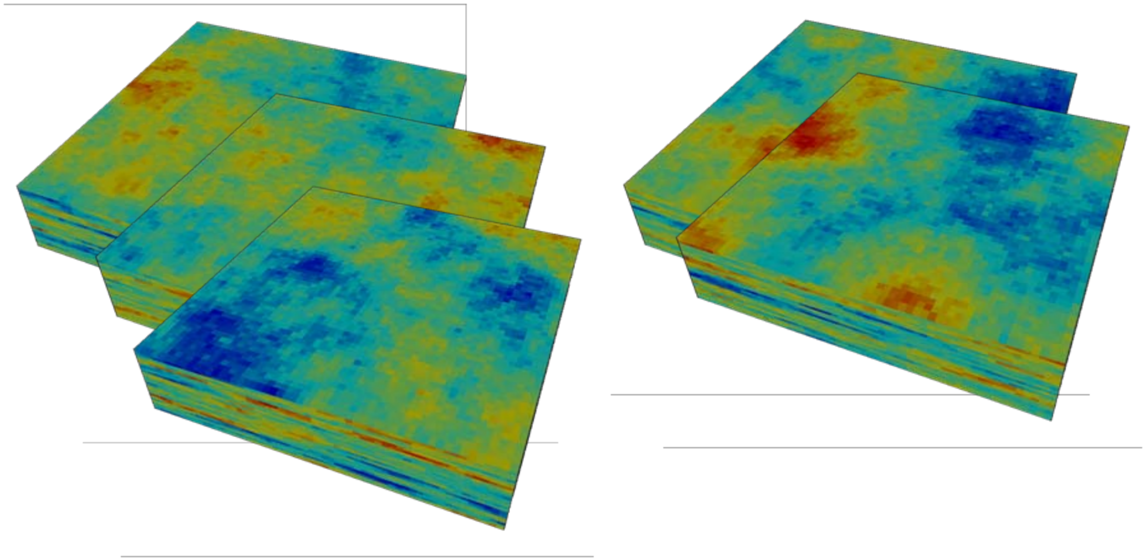
- Complications:
  - Number of wells must be determined in optimization
  - Each well can be of any type (categorical)
  - Reservoir geology is uncertain
  - Well settings also need to be optimized
  - Multiple objectives may be important
  - Each function evaluation entails reservoir simulation

Some Possible Well Types …



(from TAML, 1999)

… Coupled with Multiple Geomodels



$$\langle \mathrm{NPV} \rangle = \frac{1}{N_r} \sum_{i=1}^{N_r} (\mathrm{NPV})_i$$

***N**$_r$* **= # of realizations (potentially 100s or 1000s)**

Retrospective Optimization* for Well Placement under Geological Uncertainty

- Brute force approach: at each iteration evaluate

$$\langle J \rangle = \frac{1}{N_r} \sum_{i=1}^{N_r} J_i$$

- RO approach:
  - Define sequence of sub-problems P_k with increasing N_k
  - E.g., for N_r =80, use 4 sub-problems with N_k = (4, 8, 24, 80)
  - Optimize $\langle J \rangle_k$ using any core optimization algorithm
  - Initial guess for P_{k+1} is solution to P_k
  - Early sub-problems faster to evaluate; later sub-problems converge quickly because initial guess is close to optimum

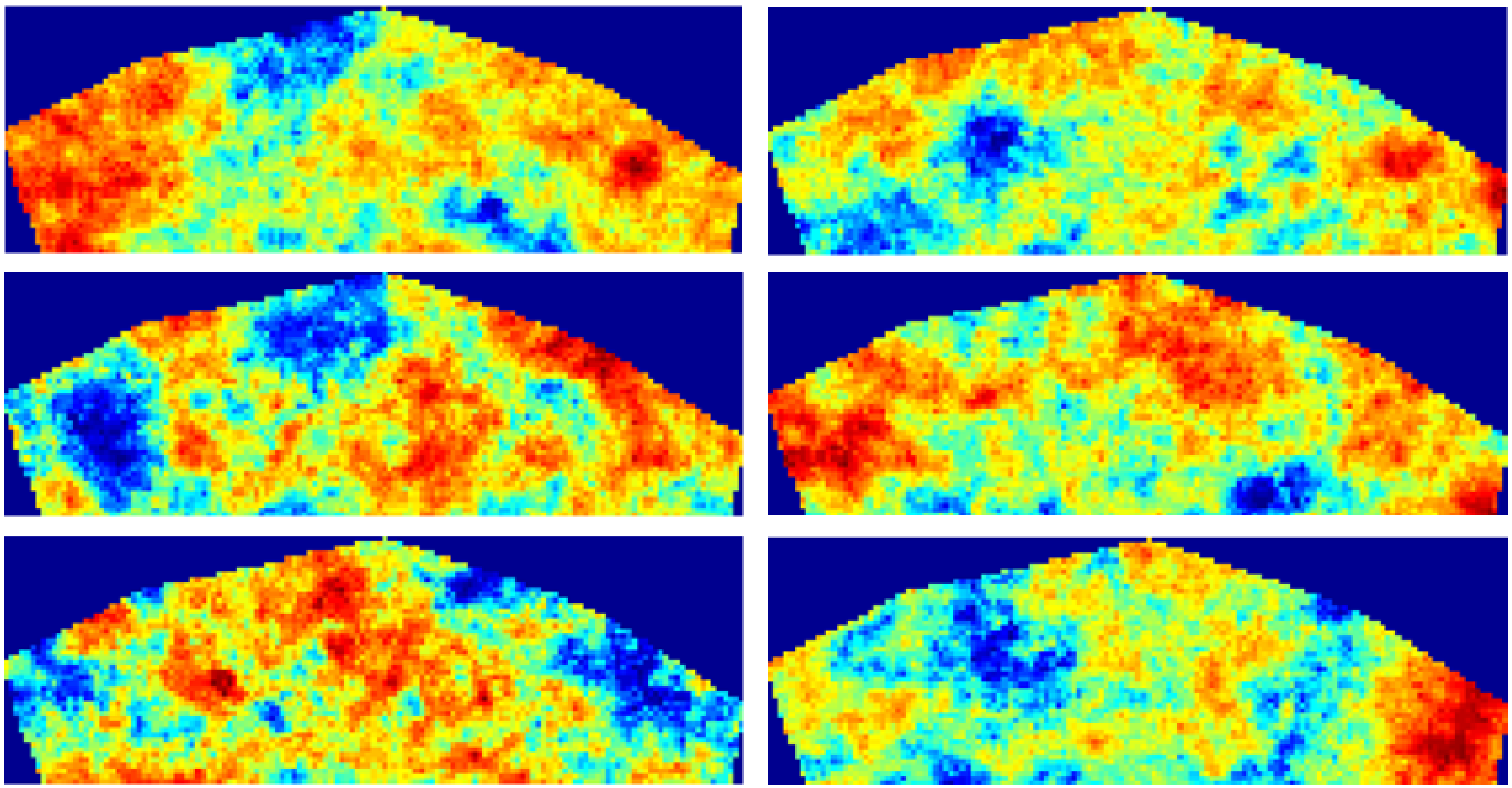
*Chen & Schmeiser, 2001; Wang & Schmeiser, 2008

Case 1: Well Placement in Brugge Field*

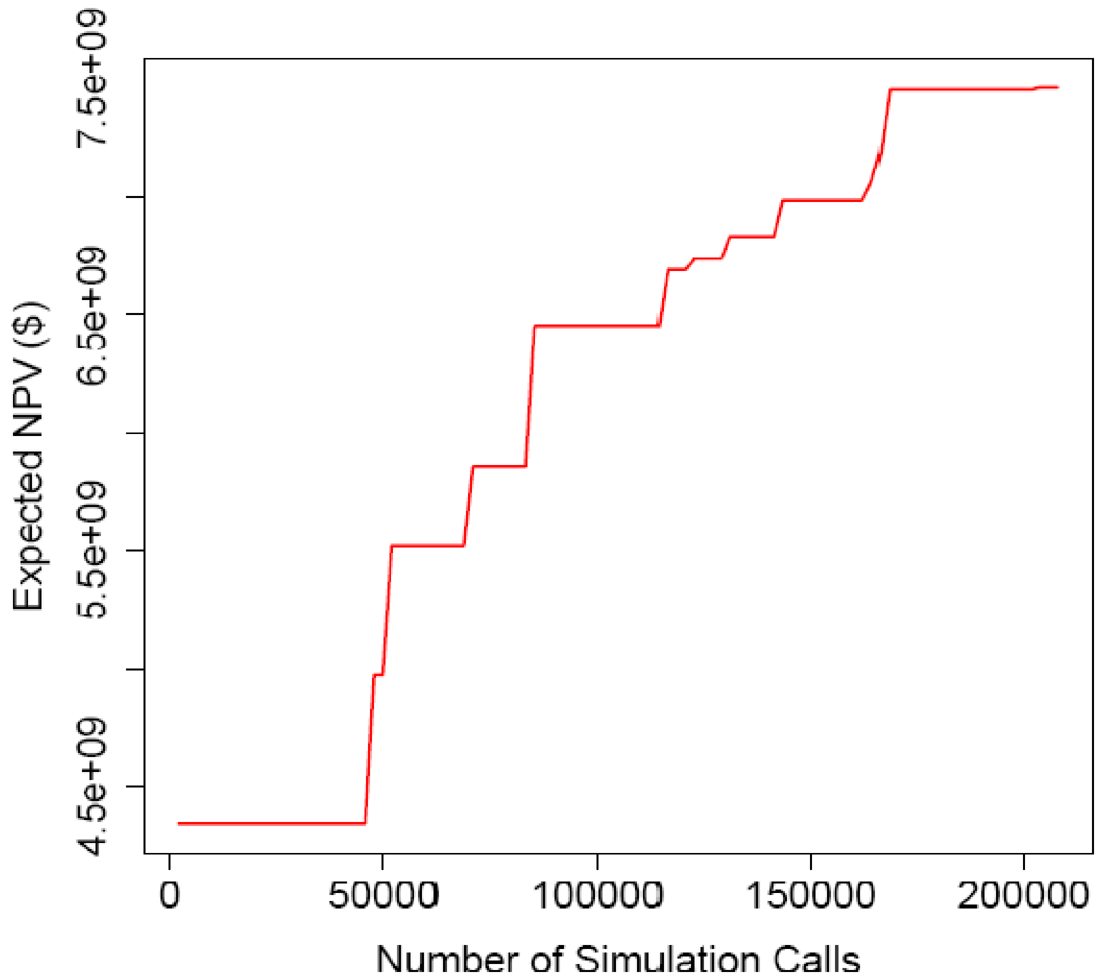
- $139 \times 48 \times 9$ blocks (total of 60,048)
- 5 fixed injection wells (BHP = 180 bar)
- Optimize 5 production wells (I, J, K_1, K_2) (BHP = 50 bar)
- 30 years of production
- Maximize NPV over 104 realizations
- Simulate using Eclipse
- Optimize using PSO

*Peters et. al. SPE 119094

Six Realizations of Brugge Permeability



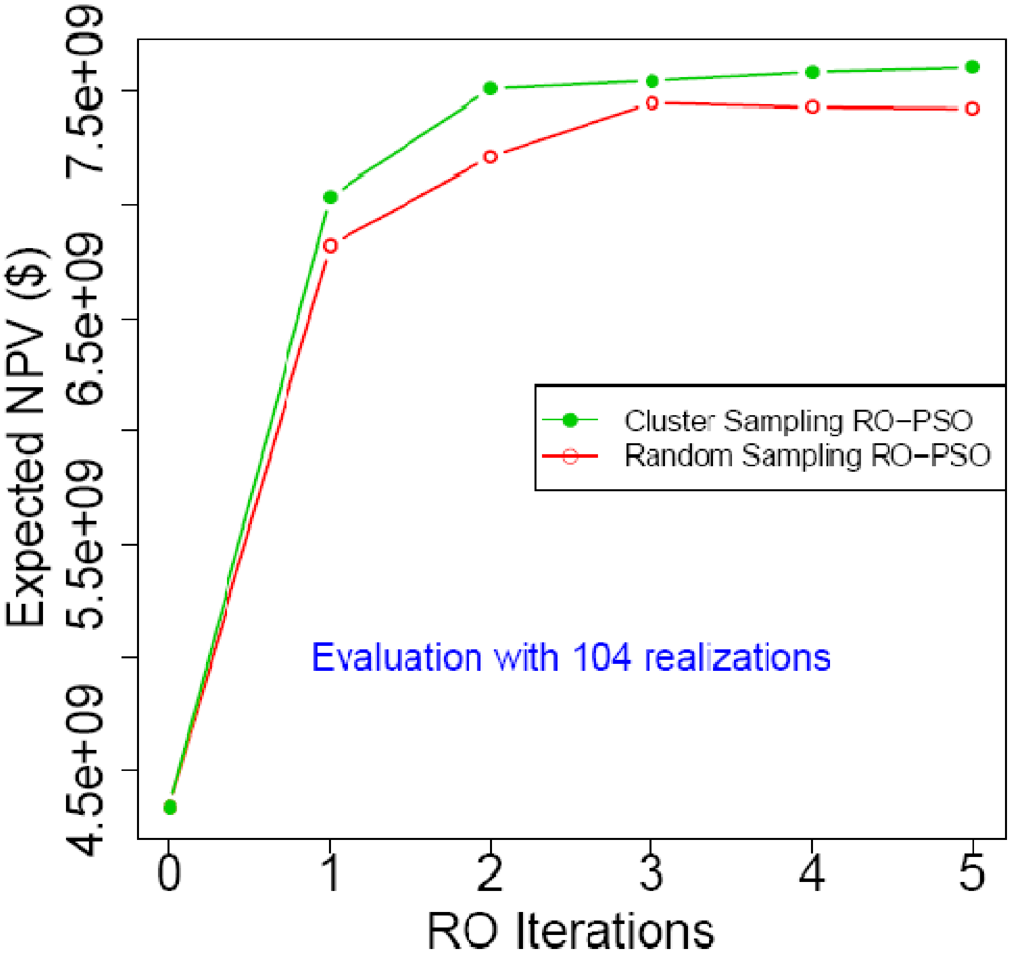
Performance of Brute-Force PSO (no RO)



- 100 PSO iterations × 20 PSO particles × 104 realizations ≈ **200,000** reservoir simulations

Wang et al. (2011)

Random and Cluster Sampling RO-PSO ($N_k = 1, 5, 16, 21, 104$, Average of 3 Runs)



PSO Full: 7.46B$
RO-PSO Random: 7.42B$
RO-PSO Cluster: 7.61B$

5 RO iterations used $\sim$ 12,000 simulations

Wang et al. (2011)

Summary

- Computational challenges:
  - Multiscale geology and effects on flow, particularly for complex (multiphysics) processes
  - Increasing importance of unconventional resources
  - Optimization of production or CO_2 sequestration
- Computational advances could lead to better predictive models, improved recovery, realistic UQ, and could facilitate production of unconventional resources