Computational Challenges for Subsurface Flow Modeling and Optimization

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Computational Issues for Oil / Gas Production and CO₂ Sequestration

- Multiscale geology
- Unconventional resources and complex recovery processes increasingly important
  - Tar sands, oil shales, shale gas, coalbed methane
  - Recovery / CO₂ storage: multiphysics & multiscale
- Use of computational optimization and data assimilation procedures; multiple objective optimization
- Uncertainty assessment, risk management
Darcy Flow through Porous Formations

- Darcy velocity \( u \), volumetric flow rate \( q \) \( (u = q/A) \)

\[
p = p_1 \quad \text{to} \quad p = p_2
\]

rock permeability \( k \), fluid viscosity \( \mu \), porosity \( \phi \)

\[
u = -\frac{k \ dp}{\mu \ dx}, \quad \phi = \frac{V_{\text{pore}}}{V_{\text{bulk}}} \]

- In multiple dimensions, \( u = -\frac{1}{\mu}k\nabla p \)
Governing Equations for Subsurface Flow (Multicomponent, Multiphase, Darcy Scale)

• Component mass balances for component \( i \):

\[
\frac{\partial}{\partial t} \left( \sum_j \phi \rho_j y_{ij} S_j \right) + \nabla \cdot \left( \sum_j \rho_j y_{ij} u_j \right) = m_c \quad n_c \text{ equations}
\]

• Darcy’s laws for phase \( j \):

\[
u_j = -\frac{k_{rj}(S)}{\mu_j} k \nabla p_j
\]

• Equilibrium relationships:

\[
K = y_{i,\text{vapor}} / y_{i,\text{oleic}}
\]

• Additional (problem specific) effects: energy equation, geomechanics, chemical reactions, …
Multiscale Geology & Reservoir Flow

- Reservoir: 5 km × 5 km × 100 m
- Grid blocks: 5 m × 5 m × 0.2 m
- \( 1000 \times 1000 \times 500 = 0.5 \times 10^9 \) blocks
Fault Zone Features

(from Ahmadov et al., 2007)
Thin Section of Slip Bands

- Low $\phi$ and $k$ in slip bands; presence of open fractures may have strong impact on flow

(from Ahmadov et al., 2007)
Upscaling Approaches

• Upscaling (computational homogenization) methods provide averaged or coarse-grained results

\[ \nabla \cdot (\lambda(S)k \cdot \nabla p) = \tilde{q}_t \]

\[ \phi \frac{\partial S}{\partial t} + \nabla \cdot (uf(S)) = -\tilde{q}_w \]

\[ \nabla \cdot (\lambda^*(S^c)k^* \cdot \nabla p^c) = \tilde{q}_t \]

\[ \phi^* \frac{\partial S^c}{\partial t} + \nabla \cdot (u^c f^*(S^c)) = -\tilde{q}_w \]

course functions \((k^*, \phi^*, \lambda^*, f^*)\) generally precomputed
Multiscale Finite Element / Volume Methods

- Construct (local) basis functions to capture subgrid $k(x)$
- Reconstruct fine-scale $u$ for transport calculations
- **FE** (Hou, Wu, Efendiev, …), **FV** (Jenny, Lee, Tchelepi, Lunati, …), **MFE** (Arbogast, Aarnes, Krogstad, Lie, Juanes, Chen …)

(figures from Hesse, Mallison, Tchelepi)
Reduced-Order Modeling Procedures

- ROM approaches require “training” run using full-order model \( [g(x)=0] \) and basis construction

- “Snapshot” matrix \( X = [x_1, x_2, \ldots, x_N] \); SVD of \( X \) provides basis matrix \( \Phi \) (\( x=\Phi z \))

- Basic POD: \( J\delta = -g \rightarrow (\Phi^T J\Phi)\delta_r = -\Phi^T g \)

- Trajectory piecewise linearization (TPWL):

\[
\begin{aligned}
    z^{n+1} &= z^{i+1} - (J_r^{i+1})^{-1} \left[ \left( \frac{\partial g^{i+1}}{\partial x^i} \right)_r (z^n - z^i) + \left( \frac{\partial g^{i+1}}{\partial u^{i+1}} \right)_r (u^{n+1} - u^{i+1}) \right] \\
    J_r^{i+1} &= \Phi^T J^{i+1} \Phi
\end{aligned}
\]
Oil from Unconventional Sources

• Alberta tar sands: $\sim 170 \times 10^9$ bbl in reserves
• US oil shales: $\sim 2 \times 10^{12}$ bbl resource (Green River Formation, UT, CO, WY)

pictures from http://www.technologyreview.com/NanoTech/wtr_16059,318,p1.html (left), S. Graham (right)
In-situ Upgrading of Oil Shale

kerogen(s) → oil + gas
(via several reactions)
In-situ Upgrading Simulations

Fan, Durlofsky & Tchelepi (2009)
Geological Sequestration of CO₂

(from Benson, 2007)
Geologic Storage of Carbon

Trapping Mechanisms

- Structural
- Residual
- Solubility
- Mineral

(from Benson, 2007)
Optimization of Well Placement and Settings

• Determine well locations, orientation and injection schedule to minimize mobile CO₂

• Applying global stochastic search (PSO) and local search (GPS, HJ) optimizers

(work with David Cameron)
Optimization Results

Optimal Injection Strategy (4 periods)

<table>
<thead>
<tr>
<th>Well 1</th>
<th>Well 2</th>
<th>Well 3</th>
<th>Well 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Injection Rate</td>
<td>Injection Rate</td>
<td>Injection Rate</td>
<td>Injection Rate</td>
</tr>
<tr>
<td>Default</td>
<td>Mobile CO$_2$ (top view, vertically averaged, low hysteresis)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Mobile CO$_2$
(top view, vertically averaged, low hysteresis)

Default

Optimized
Optimization of Field Development

• Goal is, e.g., to maximize net present value of multi-well development project

• Complications:
  – Number of wells must be determined in optimization
  – Each well can be of any type (categorical)
  – Reservoir geology is uncertain
  – Well settings also need to be optimized
  – Multiple objectives may be important
  – Each function evaluation entails reservoir simulation
Some Possible Well Types …

- Stacked Dual and Tri-Lateral
- Dual-Opposed Lateral and Stacked Opposed Quadrilateral
- Planar Dual-lateral or planar Y-well
- Planar Tri-Lateral
- Planar Offset Quadrilateral
- Planar Opposed Quadrilateral or “Herring-Bone Pattern”
- Stacked/Inclined Tri-Lateral
- Radial Quadrilateral
- Radial Tri-lateral Extending from a primary vertical wellbore
- Stacked Radial Quadrilateral

(from TAML, 1999)
... Coupled with Multiple Geomodels

\[
\langle \text{NPV} \rangle = \frac{1}{N_r} \sum_{i=1}^{N_r} (\text{NPV})_i
\]

\(N_r = \# \text{ of realizations (potentially 100s or 1000s)}\)
Retrospective Optimization* for Well Placement under Geological Uncertainty

• Brute force approach: at each iteration evaluate

\[
\langle J \rangle = \frac{1}{N_r} \sum_{i=1}^{N_r} J_i
\]

• RO approach:
  – Define sequence of sub-problems \( P_k \) with increasing \( N_k \)
  – E.g., for \( N_r = 80 \), use 4 sub-problems with \( N_k = (4, 8, 24, 80) \)
  – Optimize \( \langle J \rangle_k \) using any core optimization algorithm
  – Initial guess for \( P_{k+1} \) is solution to \( P_k \)
  – Early sub-problems faster to evaluate; later sub-problems converge quickly because initial guess is close to optimum

*Chen & Schmeiser, 2001; Wang & Schmeiser, 2008
Case 1: Well Placement in Brugge Field*

- 139 × 48 × 9 blocks (total of 60,048)
- 5 fixed injection wells (BHP = 180 bar)
- Optimize 5 production wells \((l, J, K_1, K_2)\) (BHP = 50 bar)
- 30 years of production
- Maximize NPV over 104 realizations
- Simulate using Eclipse
- Optimize using PSO

*Peters et. al. SPE 119094
Six Realizations of Brugge Permeability
Performance of Brute-Force PSO (no RO)

- 100 PSO iterations \( \times \) 20 PSO particles \( \times \) 104 realizations
  \( \approx \) 200,000 reservoir simulations

Wang et al. (2011)
Random and Cluster Sampling RO-PSO
\( (N_k = 1, 5, 16, 21, 104, \text{ Average of 3 Runs}) \)

Evaluation with 104 realizations

PSO Full: 7.46B$
RO-PSO Random: 7.42B$
RO-PSO Cluster: 7.61B$

5 RO iterations used $\sim$ 12,000 simulations

Wang et al. (2011)
Summary

• Computational challenges:
  – Multiscale geology and effects on flow, particularly for complex (multiphysics) processes
  – Increasing importance of unconventional resources
  – Optimization of production or CO₂ sequestration

• Computational advances could lead to better predictive models, improved recovery, realistic UQ, and could facilitate production of unconventional resources