The role of mathematics in glaciology: Ice sheet modeling and inverse methods

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- Some background on ice sheet models
- The Parallel Ice Sheet Model (PISM)
- The Blatter-Pattyn equations
- Inverse methods
- Conclusions

Outline

Equations for ice flow

- Navier-Stokes equations without accelerations $\rightarrow$ Stokes Flow
- Viscosity of ice is stress-dependent, *non-linear Stokes Flow*
- Approximations based on shallowness (height/width $\ll 1$) $\rightarrow$ Shallow Ice Approximation (SIA)
- The SIA can be vertically integrated, flow is essentially a 2D problem
- Ice viscosity is temperature dependent, thermo-mechanical coupling

The Shallow Ice Approximation

- A good approximation of ice flow over most of an ice sheet
- Has been implemented for several decades $\rightarrow$ legacy code
- Model performance has been evaluated in intercomparison projects (Eismint, etc.)

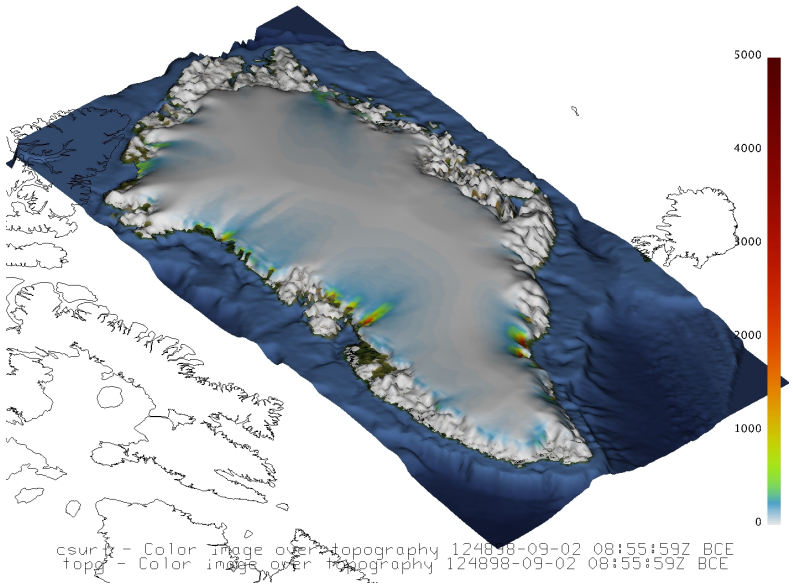
The Shallow Shelf Approximation

- A second low-order approximation has been proposed for rapidly flowing ice (MacAyeal)
- This approximation also allows vertical integration and results in an essentially 2D model
- The model has been validated against data and used in parameter inversions
- It applies in areas of fast flow and ice shelves, mostly near the margins of ice sheets

Outline

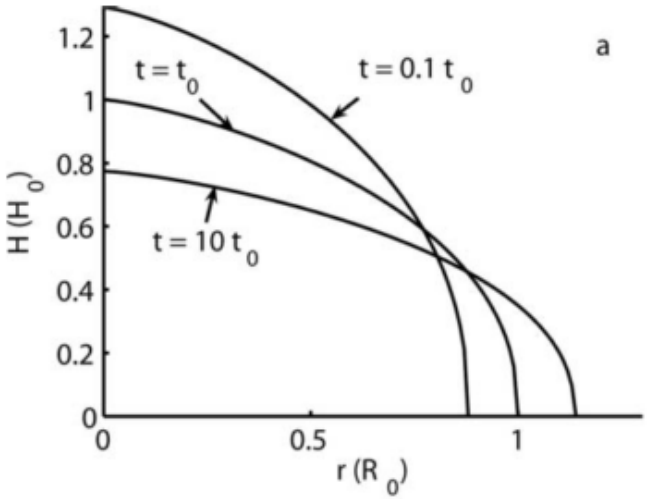
Some features of PISM

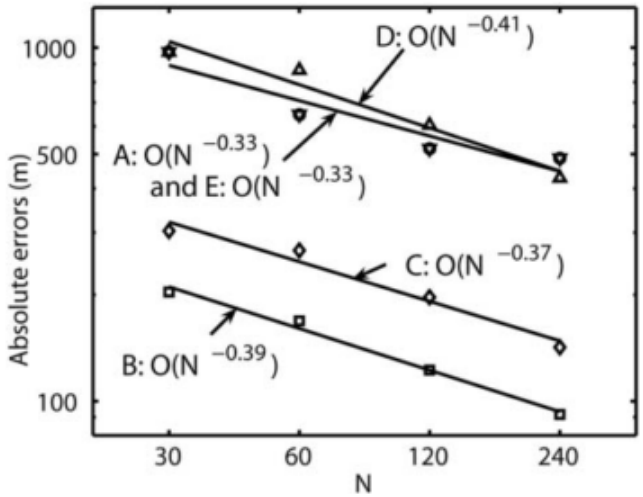
- Hierarchy of available stress balances
- Verification and validation tools
- Polythermal, enthalpy-based conservation of energy scheme
- Marine ice sheet physics, dynamic calving fronts
- Uses MPI and PETSc for parallel simulations
  - Extensible coupling to atmospheric and ocean models
  - Complete documentation for users and developers
  - Reads and writes CF 1.4-compliant NetCDF



Model verification

- *Bueler et al. (2005, 2007)* provided analytical (manufactured) solutions to the time-dependent SIA
- Verification against exact solution is a standard feature of PISM





Algorithms for ice streams

- Time-independent theory of ice streaming over plastic till (*Schoof, 2006, A variational approach to ice stream flow*)
- Implementation in time-dependent ice sheet model (*Bueler and Brown, The Shallow Shelf Approximation as a sliding law in a thermomechanically coupled ice sheet model*)

Outline

Higher-order models

- Blatter (1995) developed a first-order theory for ice flow
- Collaborators in the Mathematics dept. (Univ. Geneva, EPF Lausanne) helped implement the method
- Follow-up work addressed well-posedness and existence and uniqueness of solutions for first-order models

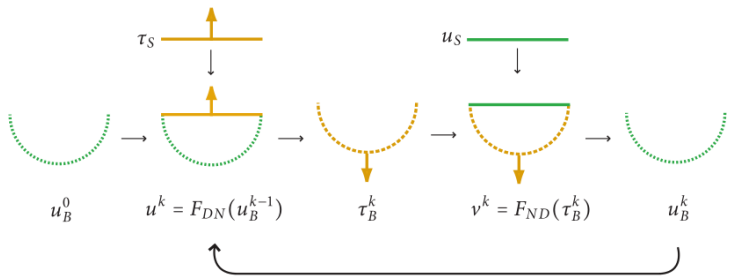
Outline

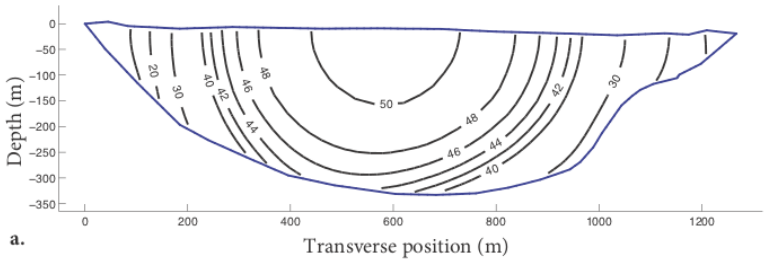
Reconstruction of the basal boundary condition

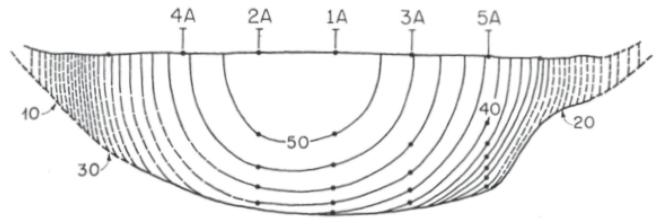
- Generally, observations are limited to the surface of a glacier
- This leads to an ill-posed problem (common in geophysics)
- Linear inverse theory is applicable in some simple situations (e.g. Truffer, 2004)

Iterative inverse methods

- Surface has Dirichlet (velocity) and Neumann (stress) boundary conditions
- Neither one of these is known at the base
- Assume basal velocities and then iterate between Dirichlet-Neumann and vice versa (*Kozlov-Maz'ya*)
- *Maxwell et al. (2008)* developed and implemented an accelerated version
- Application to Perito Moreno Glacier and in follow-up papers
- Shown equivalency to Landweber iteration







Current work

- Incorporate iterative inverse methods into PISM
- Developed a new rapidly converging method (incomplete Gauss-Newton)
- Developed inverse library (SIPL)
- All user interfacing is in Python
- Goal: initiate ice sheet models with basal boundary conditions derived from observations

Outline

Conclusions

- Geophysics-Mathematics collaboration has led to several improvements in ice-sheet models and application of inverse methods, namely
  - Verification and validation tools
  - Robust numerical analysis
  - Integration of inverse method tools for boundary conditions