Alistair Adcroft (Princeton Univ.)

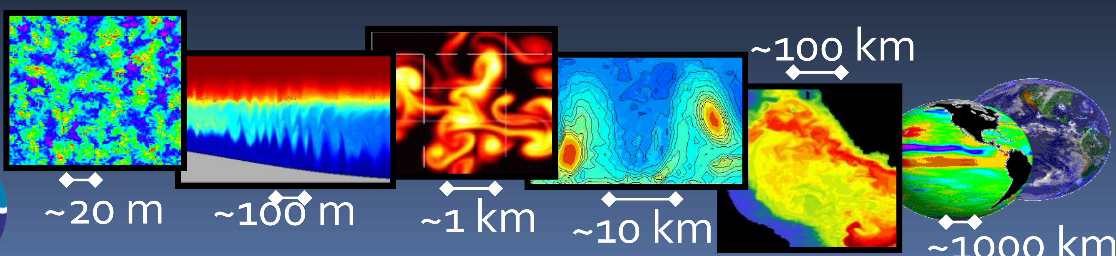
HIGH ORDER REMAPPING, THE SPURIOUS MIXING PROBLEM, AND HEAT UPTAKE IN OCEAN MODELS

White & Adcroft, JCP 2008
White et al., JCP 2009
Ilicak et al., OM 2011

On ocean climate modeling

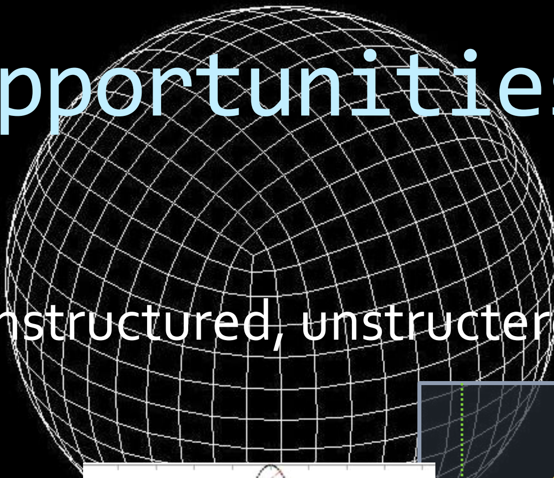
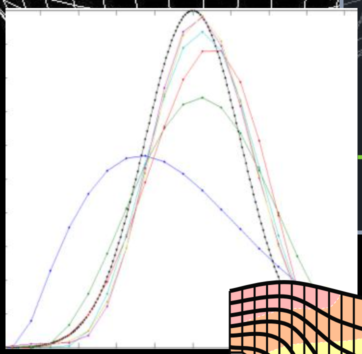
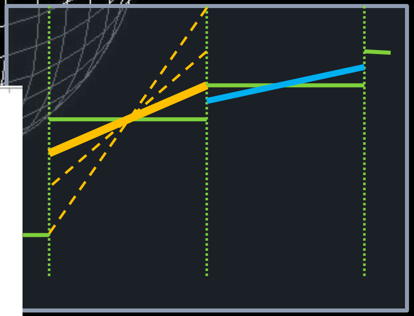
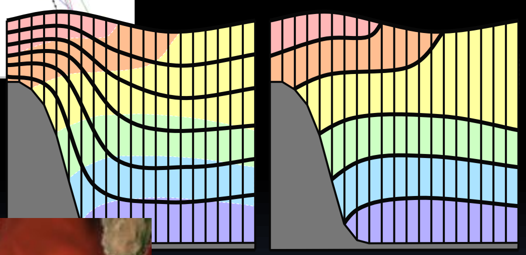
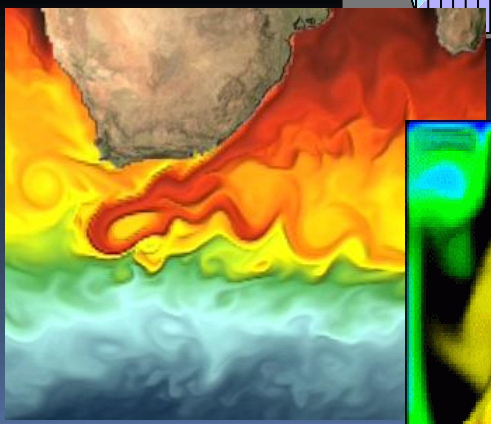
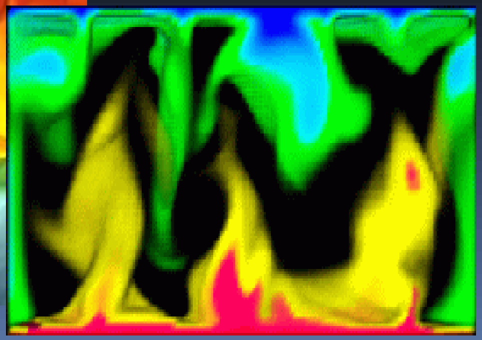
- The global ocean …
  - … is stratified
  - … is rotating
  - … has very complex boundaries
  - … has wide scale separations
- … makes modeling very challenging
- Climate problem is *not* an initial value problem
- Compared to CFD codes OGCMs can "*appear* antiquated"
  - Different computational demands
  - Turnaround trumps accuracy

- Length scales
  - Planetary/basin ~ 10^7 m
  - Mesoscale ~ 10^3-10^5 m
  - Depth ~ 10^0-10^4 m
  - Ozmidov ~ 10^0-10^2 m
  - Boundary layer ~ 10^{-2}-10^1 m
  - Kolmogorov ~ 10^{-3} m
- Time scales
  - Carbon ~ 10^{11}-10^{12} s
  - Thermal (MOC) ~ 10^{10} s
  - Eddy turn-over ~ 10^5 s
  - Rotation ~ 10^4 s
  - Forced convection ~ 10^3 s
- 1° model: $4{\times}10^5$ nodes, $\Delta_t \sim 3.10^3$ s
- ¼° model: 10^8 nodes, $\Delta_t \sim 10^3$ s
- 1 km dream: 10^{10} nodes, $\Delta_t \sim 40$ s



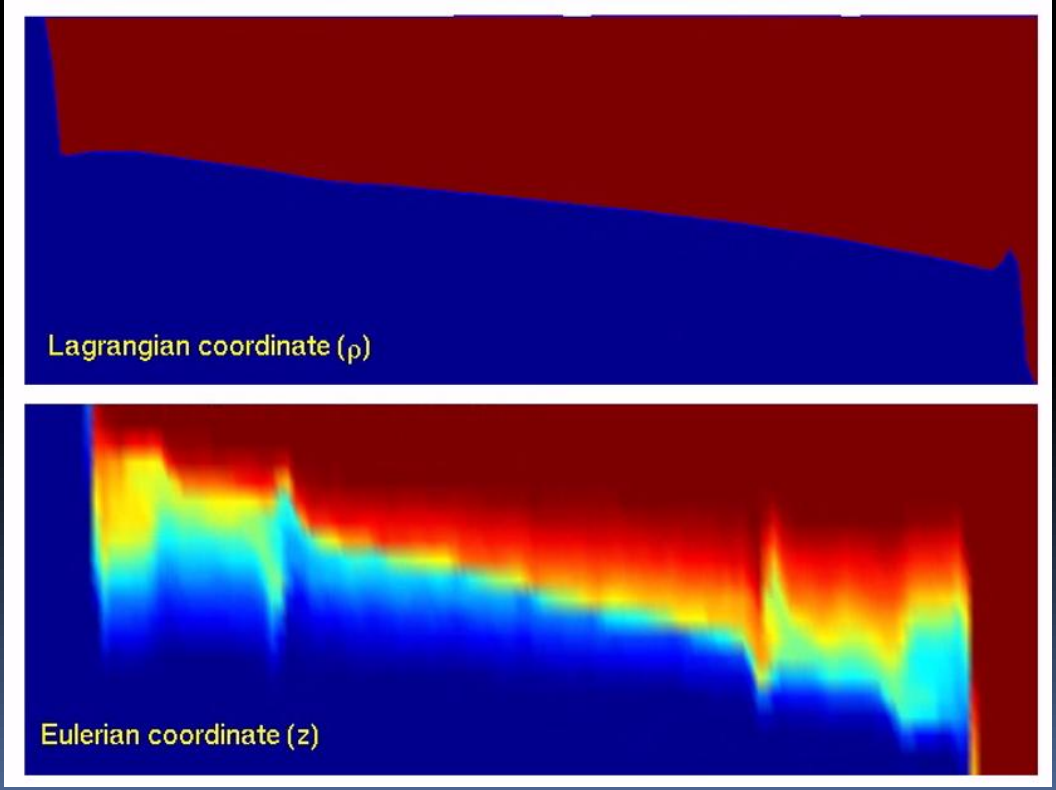
Recent advances/opportunities

- Horizontal grids
  - Stretching, nesting, macro-unstructured, unstructered grids
- Numerical methods
  - Time-stepping, solvers
- Transport schemes
  - High order, compact, limiters
- Formulation
  - Equations, symmetries, coordinates
- Data assimilation
- Parameterizations
  - S.G.S. turbulent fluxes
    - Momentum
    - Heat/salt/BGC scalars

Example of spurious mixing

- Numerical truncations exhibited as diffusion (and dispersion) *spuriously* lead to new water masses



"True" solution: Isopycnal models preserve water masses in the adiabatic limit

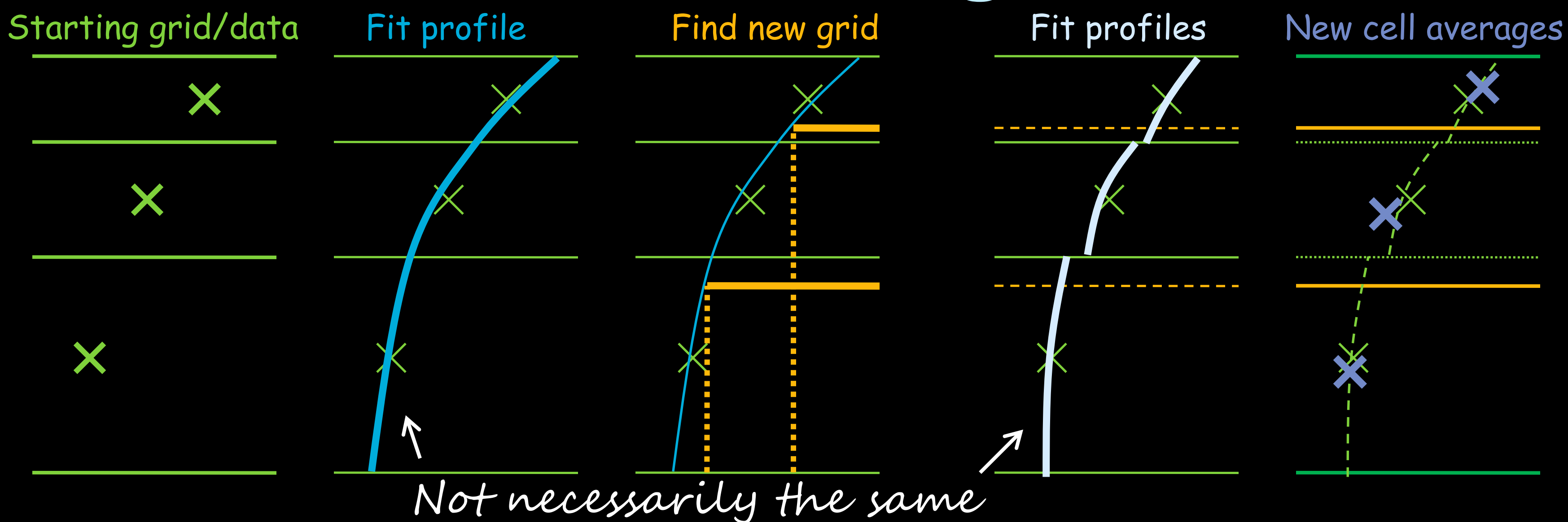
"New" density classes appear due to motion normal to the isopycnal

Ilicak et al., OM 2011

Spurious mixing

- Spurious *diapycnal* mixing Griffies et al, 2000
  - One argument in “coordinate debate”
    - Isopycnal models can preserve water masses
  - Believed to be more prevalent at higher resolution (eddy permitting)
  - Model inter-comparisons often “compare apples and oranges” DYNAMO; Legg et al., 2006 (GCE-CPT)
    - Coordinate issue best evaluated in ***one*** model
    - New general coordinate model “GOLD”
    - New high order methods: PQM
- Diagnose spurious mixing
  - In idealized experiments
  - In “realistic” global models
- Consequences for ocean-climate models
  - Heat uptake controlled by mixing

Coordinate free algorithm

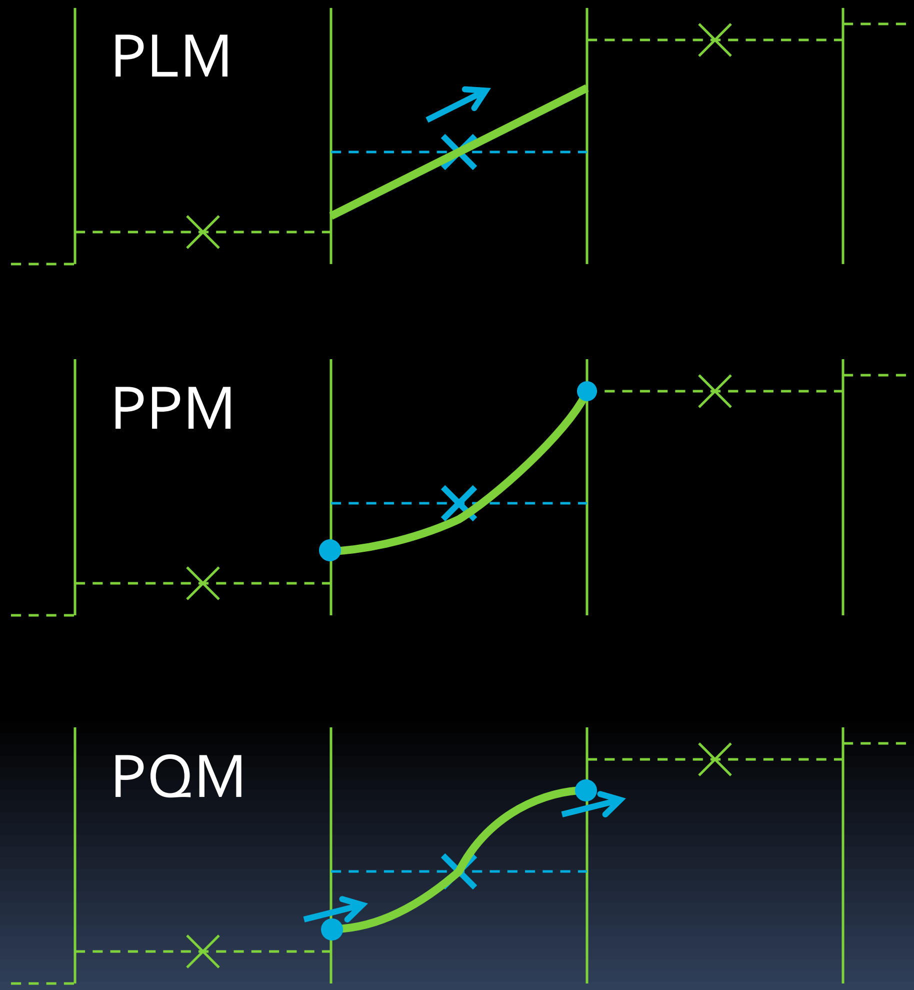


- Re-gridding
  - Re-construct **global** profile
    - **Single valued (monotonic)**
    - (continuous or not)
    - (conservative or not)
  - Find position of new grid
- Re-mapping
  - Re-construct **local** profiles
    - **Conservative**
    - Limited (monotonic) or not
    - Discontinuous (exclusive!) or not
  - Integrate for new cell averages

- <u>***Accuracy of reconstructions***</u> is key to success of remapping algorithms

Piecewise * Method (* = C,L,P or Q)

- PLM: two degrees of freedom
  - Cell mean + slope
- PPM: three degrees of freedom
  - Very widely used
  - Cell mean + two edge values
- PQM: five degrees of freedom
  - Cell mean + two edge values + two edge slopes



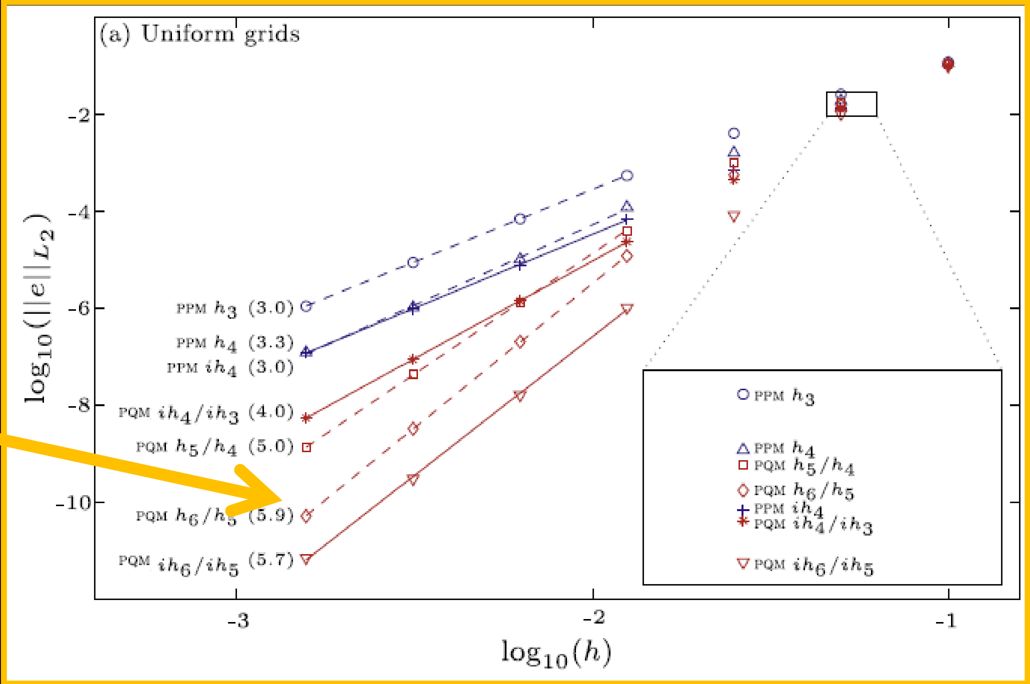
Successive schemes provide more flexibility to represent structures → more accurate

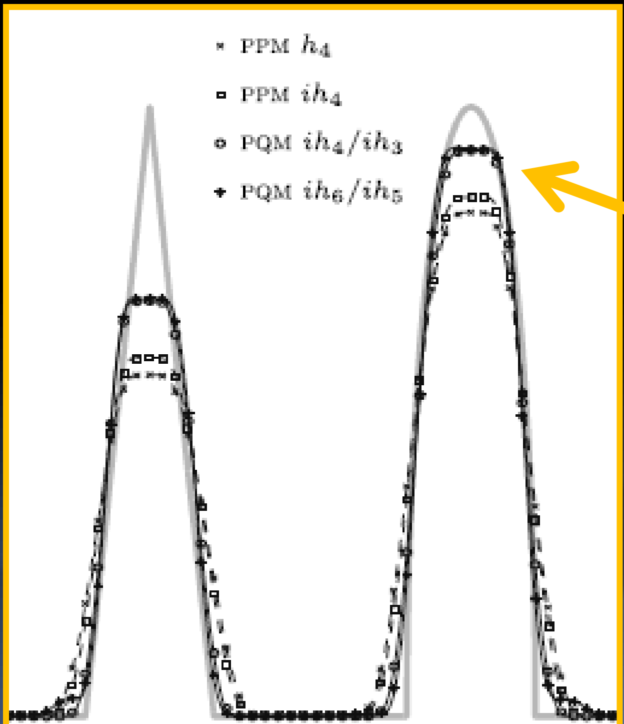
White & Adcroft, JCP 2008

PQM results

- 1D advection test
- 5th order accuracy

  (or better)
  - when unlimited





- PQM substantially more accurate than PPM in practice
  - i.e. when limited

    (as expected)

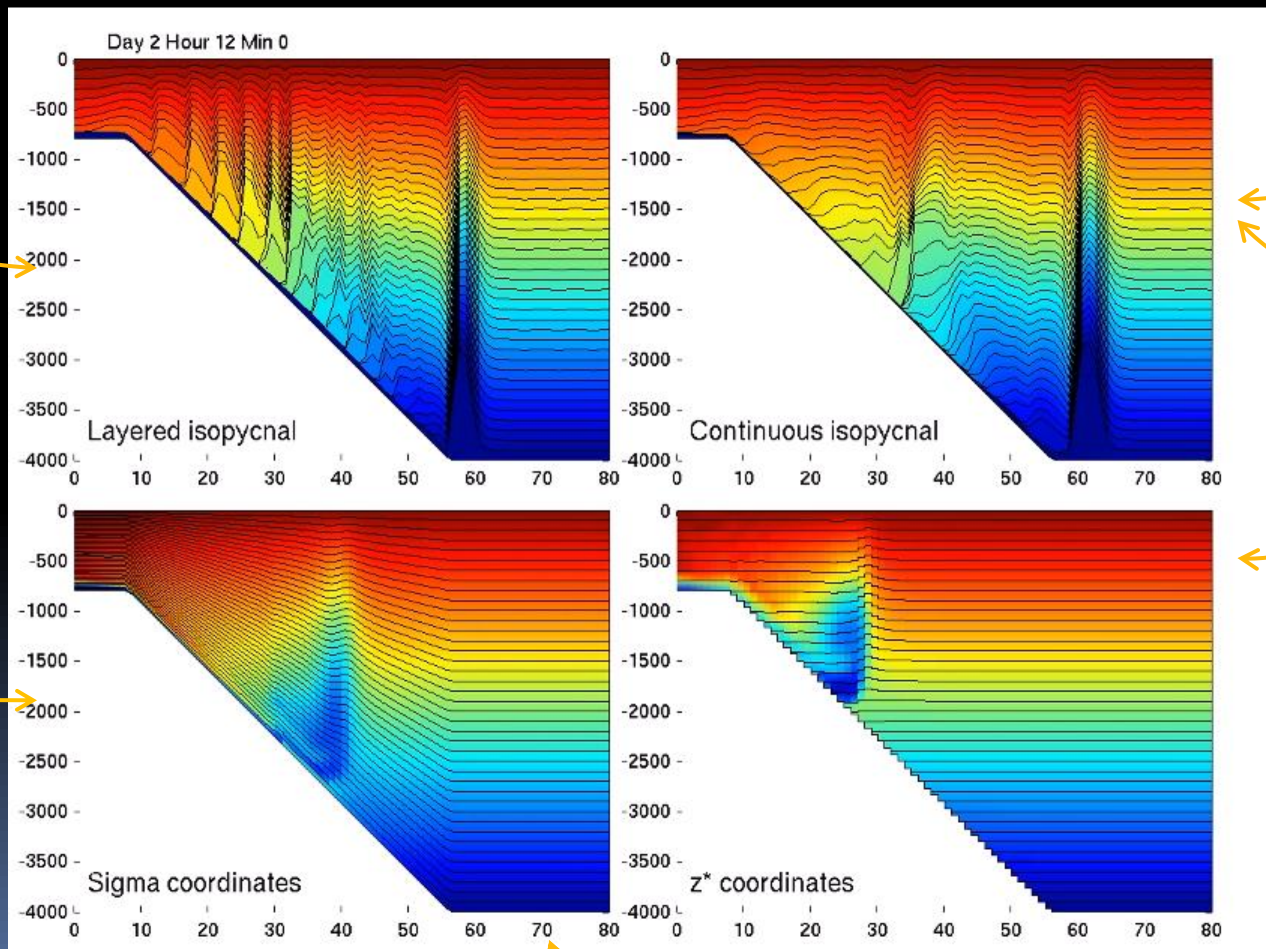
White & Adcroft, JCP 2008

Assessing coordinates in **one** model

- Spurious diffusion significantly dilutes gravity current
- Re-mapping to isopycnals does as well as layered isopycnal
- Re-mapping to <u>non-isopycnal coordinates clearly diffusive</u>

"True" sol^{n} (adiabatic)

Z* and σ dillute buoyancy anomaly



Better sol^{n} :-)

Same numerics for non-layered models

White et al., JCP 2009

Quantifying spurious mixing

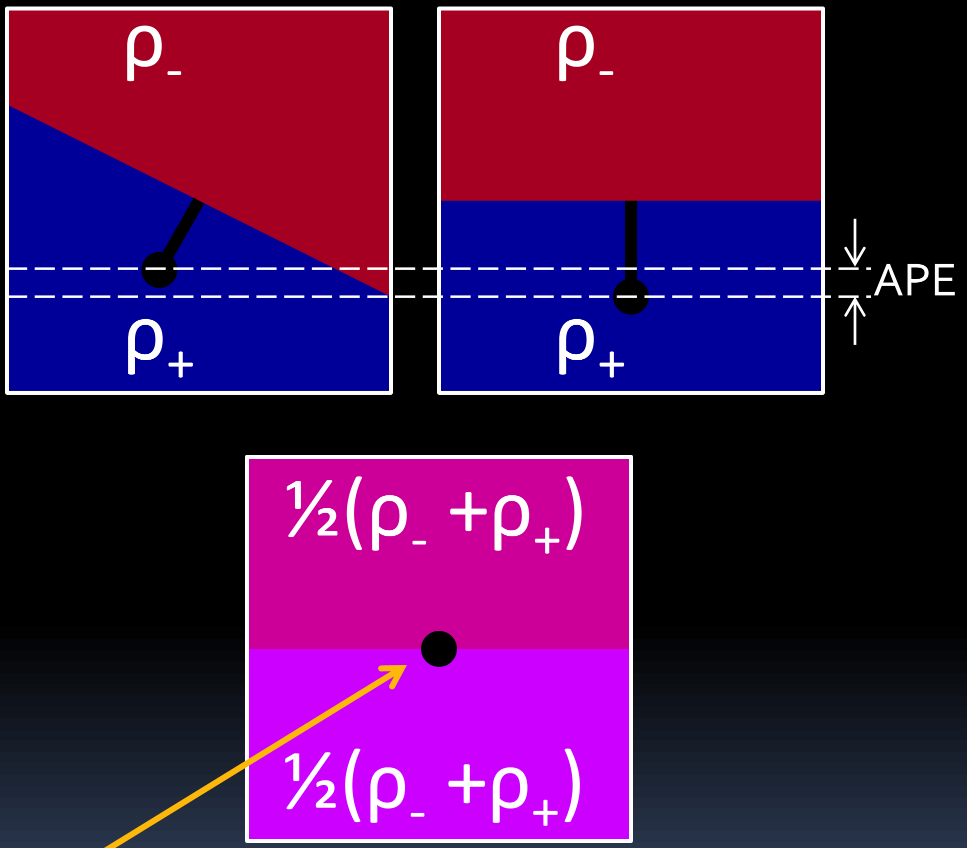
- Potential energy

$$PE = g\iiint \rho z \, dV$$

- Available potential energy (APE)

$$APE = PE - UPE$$
$$UPE = g\iiint \rho^* z \, dV$$

- ρ* is the adiabatically re-arranged state with minimal potential energy
- UPE can only be changed by diapycnal mixing
  - Mixing raises center of mass



Winters et al., JFM 1995
Ilicak et al., OM 2011

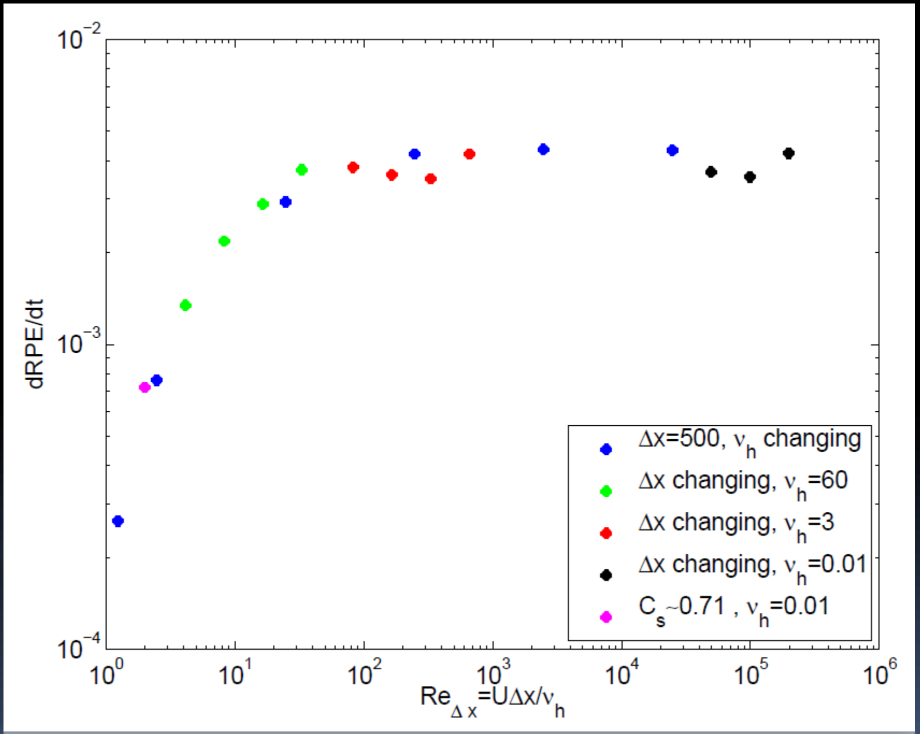
What controls spurious mixing

1. Accuracy of transport scheme most significant at low orders
   - Large difference between 1^{st} and 2^{nd} order
   - Small difference between 3^{rd} and 7^{th} order
2. Noise in flow field
   - Controlled by grid Reynolds number

$$\mathrm{Re}_{\Delta} = \frac{U\Delta x}{\nu_h}$$

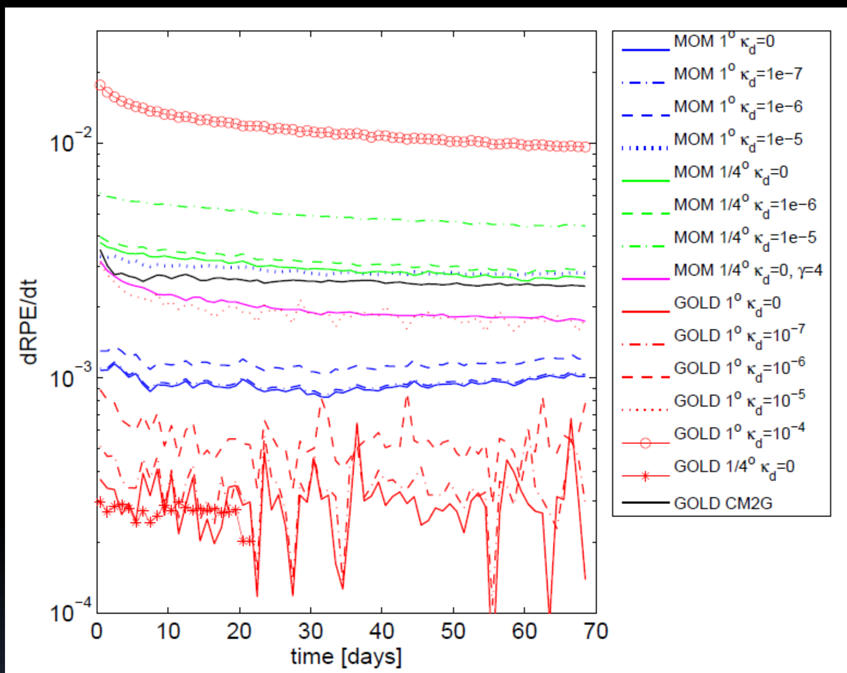
   - Usual practice is to use largest Re_{Δ} that is stable!

Note: this concerns 3D transport in non-isopycnal coordinates
Isopycnal coordinates have *negligible* spurious mixing



Ilicak et al., OM 2011

Spurious work in global models I



- Spin down experiments
- Realistic configurations
  - Geometry
  - Spun-up hydrography
  - Momentum closure
- Measure real energy change
  - Due only to spurious or explicit mixing

Ilicak et al., OM 2011

Spurious work in global models II

- Intended work by parameterized diapycnal processes in CM2G (1°) does 916 GW
  - **809 GW** adjusted for cabbelling
  - Corresponds to ~3 TW wind/tide input (whole ocean)

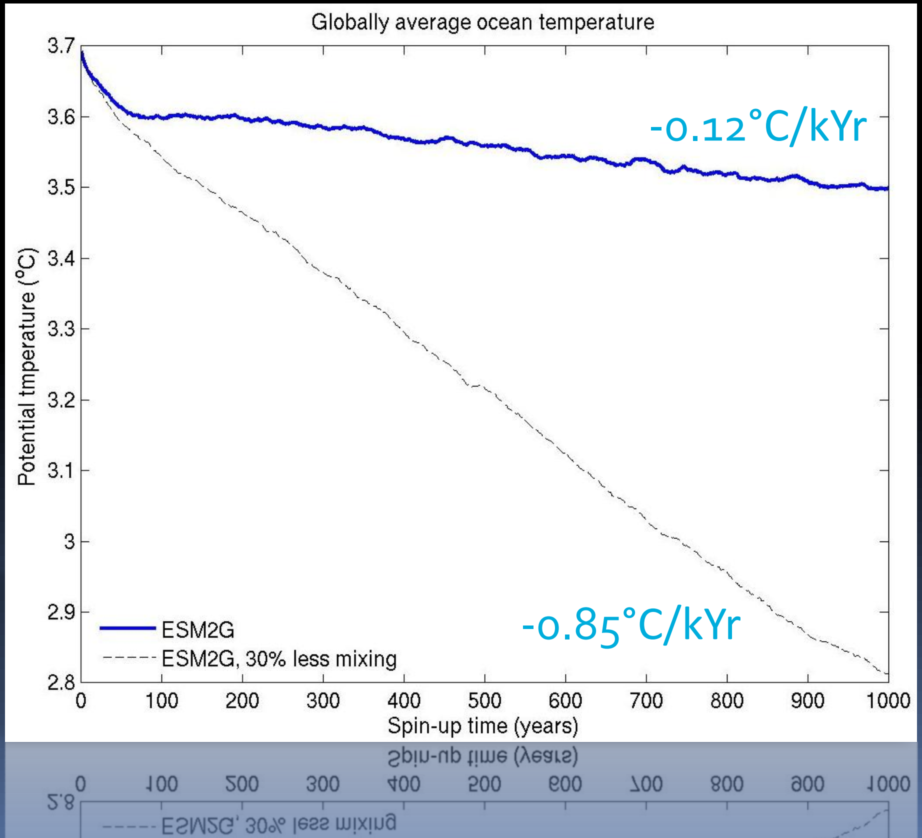
κ (m^2/s)	ρ-coord 1° (GW)		z-coord 1° (GW)		z-coord ¼° (GW)	
0	**107**	0	**336**	0	**1015**	0
10^{-7}	124	17	344	8	-	-
10^{-6}	192	85	411	75	1089	74
10^{-5}	656	549	1012	676	1701	686
10^{-4}	3819	3712	-	-	-	-

Would like these to be small compared to 809 GW

Ilicak et al., OM 2011

Heat uptake in ocean models

- Abyssal mixing matters for long term climate
  - Affects long term heat uptake
  - … and for carbon (closed system)



Due to increasing parameterized mixing
(~30% more work)

Final thoughts

- Improvements in numerical methods are paying off
  - Enables new class of models to address questions that couldn't be answered before
- Applying modern methods is hard
  - Different constraints from other CFD problems
  - Different computational demands
    - Direction of computer evolution is not helping