Seismic Imaging based on Spectral-element and Adjoint Methods

Jeroen Tromp
(CMG collaboration with Ingrid Daubechies)

CMG workshop 2011

Forward Problem in Seismology

Equation of motion:

$$\rho\,\partial_t^2\mathbf{s} - \boldsymbol{\nabla}\cdot\mathbf{T} = \mathbf{f}$$

Constitutive relationship (may be anelastic):

$$\mathbf{T} = \mathbf{c} : \boldsymbol{\epsilon}$$

Boundary condition:

$$\hat{\mathbf{n}}\cdot\mathbf{T} = \mathbf{0}$$

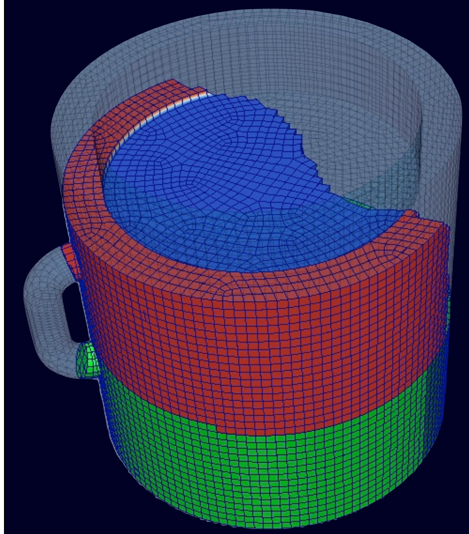
Initial conditions:

$$\mathbf{s}(\mathbf{x},0) = \mathbf{0}, \qquad \partial_t\mathbf{s}(\mathbf{x},0) = \mathbf{0}$$

Earthquake source:

$$\mathbf{f} = -\mathbf{M}\cdot\boldsymbol{\nabla}\delta(\mathbf{x}-\mathbf{x}_{\mathrm{s}})S(t)$$

Spectral-Element Simulations



Open Source software: SPECFEM3D_SESAME

Dimitri Komatitsch, Daniel Peter

Adjoint Tomography

PDE-constrained waveform tomography:

$$\chi = \frac{1}{2}\sum_r \int_0^T [\mathbf{s}(\mathbf{x}_r,t) - \mathbf{d}(\mathbf{x}_r,t)]^2 \,\mathrm{d}t - \int_0^T \int_\Omega \boldsymbol{\lambda}\cdot(\rho\,\partial_t^2\mathbf{s} - \boldsymbol{\nabla}\cdot\mathbf{T} - \mathbf{f})\,\mathrm{d}^3\mathbf{x}\,\mathrm{d}t$$

Change in the waveform misfit function:

$$\begin{aligned}
\delta\chi = &\int_0^T \int_\Omega \sum_r [\mathbf{s}(\mathbf{x}_r,t) - \mathbf{d}(\mathbf{x}_r,t)]\delta(\mathbf{x}-\mathbf{x}_r)\cdot\delta\mathbf{s}(\mathbf{x},t)\,\mathrm{d}^3\mathbf{x}\,\mathrm{d}t \\
&- \int_0^T \int_\Omega (\delta\rho\boldsymbol{\lambda}\cdot\partial_t^2\mathbf{s} + \boldsymbol{\nabla}\boldsymbol{\lambda}:\delta\mathbf{c}:\boldsymbol{\nabla}\mathbf{s} - \boldsymbol{\lambda}\cdot\delta\mathbf{f})\,\mathrm{d}^3\mathbf{x}\,\mathrm{d}t - \int_0^T \int_\Omega [\rho\partial_t^2\boldsymbol{\lambda} - \boldsymbol{\nabla}\cdot(\mathbf{c}:\boldsymbol{\nabla}\boldsymbol{\lambda})]\cdot\delta\mathbf{s}\,\mathrm{d}^3\mathbf{x}\,\mathrm{d}t \\
&- \int_\Omega [\rho(\boldsymbol{\lambda}\cdot\partial_t\delta\mathbf{s} - \partial_t\boldsymbol{\lambda}\cdot\delta\mathbf{s})]_T\,\mathrm{d}^3\mathbf{x} - \int_0^T \int_{\partial\Omega} \hat{\mathbf{n}}\cdot(\mathbf{c}:\boldsymbol{\nabla}\boldsymbol{\lambda})\cdot\delta\mathbf{s}\,\mathrm{d}^2\mathbf{x}\,\mathrm{d}t,
\end{aligned}$$

Adjoint Equations

Adjoint wavefield: $\mathbf{s}^{\dagger}(\mathbf{x},t) \equiv \boldsymbol{\lambda}(\mathbf{x},T-t)$

Adjoint equation of motion: $\rho\,\partial_t^2\mathbf{s}^{\dagger} = \boldsymbol{\nabla}\cdot\mathbf{T}^{\dagger} + \mathbf{f}^{\dagger}$

Adjoint boundary conditions: $\hat{\mathbf{n}}\cdot\mathbf{T}^{\dagger} = \mathbf{0}$

Adjoint initial conditions: $\mathbf{s}^{\dagger}(\mathbf{x},0) = \mathbf{0}, \qquad \partial_t\mathbf{s}^{\dagger}(\mathbf{x},0) = \mathbf{0}$

Adjoint source: $\mathbf{f}^{\dagger}(\mathbf{x},t) = \sum_{r=1}^{N}[\mathbf{s}(\mathbf{x}_r,T-t)-\mathbf{d}(\mathbf{x}_r,T-t)]\delta(\mathbf{x}-\mathbf{x}_r)$

Frechet derivative

The Frechet derivative may be expressed as:

$$\delta\chi = \int_\Omega (\delta\rho K_\rho + \delta\mathbf{c} :: \mathbf{K_c}) \, \mathrm{d}^3\mathbf{x} + \int_0^T \int_\Omega \mathbf{s}^\dagger \cdot \delta\mathbf{f} \, \mathrm{d}^3\mathbf{x} \, \mathrm{d}t$$

Density and elastic tensor kernels:

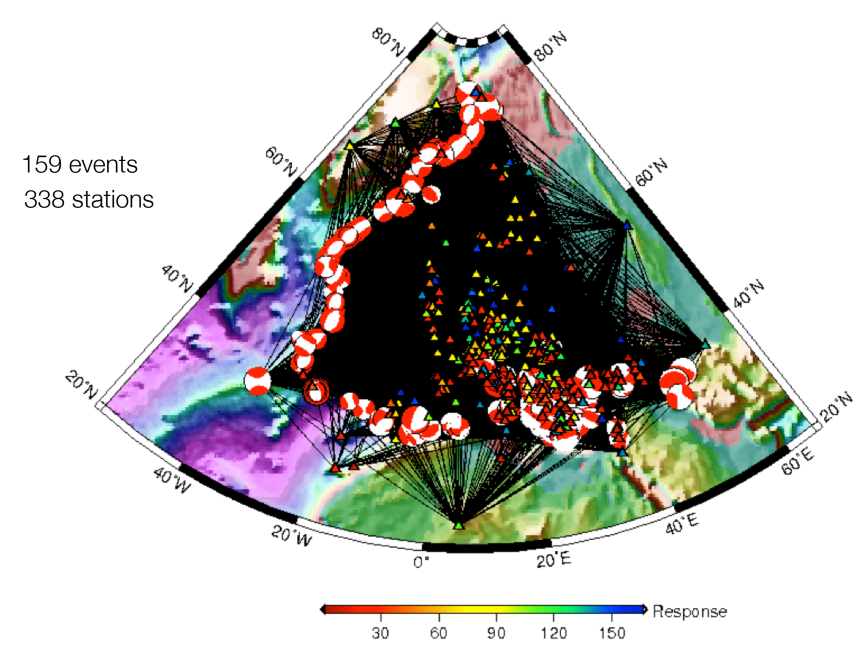
$$K_\rho(\mathbf{x}) = -\int_0^T \mathbf{s}^\dagger(\mathbf{x}, T-t) \cdot \partial_t^2 \mathbf{s}(\mathbf{x}, t) \, \mathrm{d}t$$

$$\mathbf{K_c}(\mathbf{x}) = -\int_0^T \boldsymbol{\nabla}\mathbf{s}^\dagger(\mathbf{x}, T-t) \, \boldsymbol{\nabla}\mathbf{s}(\mathbf{x}, t) \, \mathrm{d}t$$

Current & Future Research

- Model parameterzation (transverse isotropy, Q, more general anisotropy)
- Misfit
  - Measurement type (cross-correlation traveltime, frequency-dependent phase & amplitude)
  - Window selection (steadily increase number of “picks”)
  - Band pass filtering (steadily increase frequency content)
- Model basis
- Regularization/smoothing
- Preconditioning
- Conjugate-gradient algorithm
- Sampling the posterior model distribution
- Resolution analysis (Hessian kernels)
- Validation

Adjoint Tomography of Europe



PhD thesis of Hejun Zhu

Misfit Function

$$\chi(\mathbf{m}) = \frac{1}{6}\sum_{Z,R,T}\frac{1}{N_s}\sum_{s}\frac{1}{N_p}\sum_{i=1}^{p}\left(\frac{\Delta T_i}{\sigma_i}\right)^2 + \frac{1}{6}\sum_{Z,R,T}\frac{1}{N_s}\sum_{s}\frac{1}{N_p}\sum_{i=1}^{p}\int W_i\left[\frac{\Delta\tau(\omega)_i}{\sigma(\omega)_i}\right]^2 d\omega$$

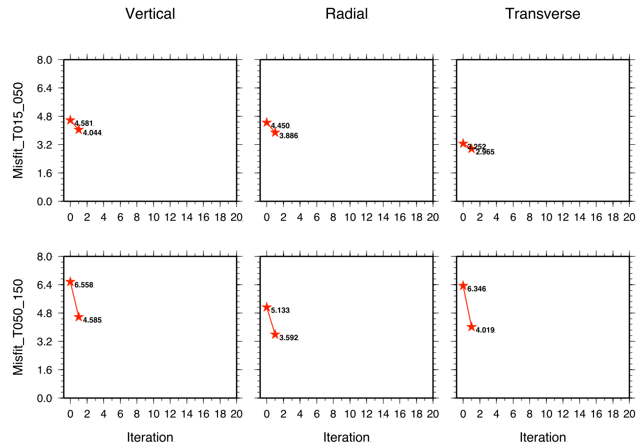
Body-wave cross correlation/multitaper
15-40 s band pass
(1) P-SV (Vertical)
(2) P-SV(Radial)
(3) SH (Transverse)

Surface-wave multitaper
25-150 s band pass
(4) Rayleigh (Vertical)
(5) Rayleigh (Radial)
(6) Love (Transverse)

Adjoint source = SP(body-wave measurement) + LP (surface-wave measurement)

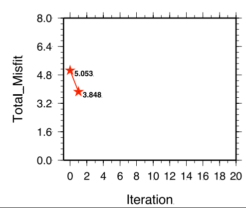
$$\delta\chi^{TI} = \int K_c\delta\ln c + K_{\beta_v}\delta\ln\beta_v + K_{\beta_h}\delta\ln\beta_h + K_\eta\delta\ln\eta + K_\rho\delta\ln\rho\, dV$$

Reduction in Misfit: First Iteration



$$\chi_c = \frac{1}{N_s}\sum_s \frac{1}{N_p}\sum_p \left(\frac{\Delta T_p}{\sigma_p}\right)^2$$

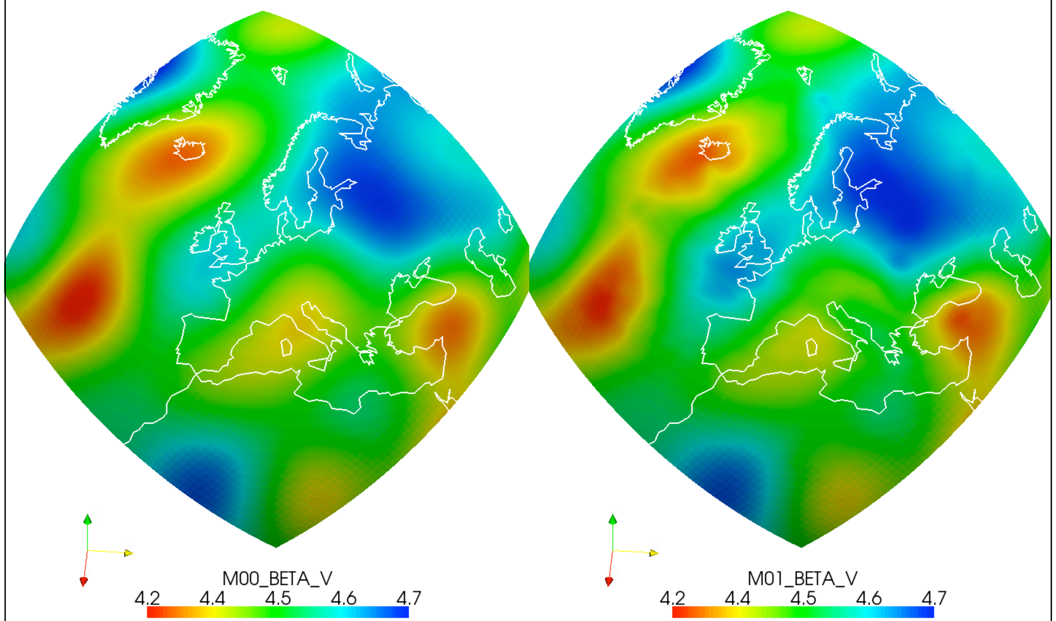
$$\chi_c = \frac{1}{N_s}\sum_s \frac{1}{N_p}\sum_p \int W_p \left(\frac{\Delta \tau_p(\omega)}{\sigma_p(\omega)}\right)^2 d\omega$$



$$\chi = \frac{1}{N_c}\sum_c \chi_c$$

- 50 minutes per simulation (168 cores)
- 3 simulations per event per iteration
- 1 iteration ~1 day
- Source + structure inversion: ~1.5M CPU hours

Depth 75 km



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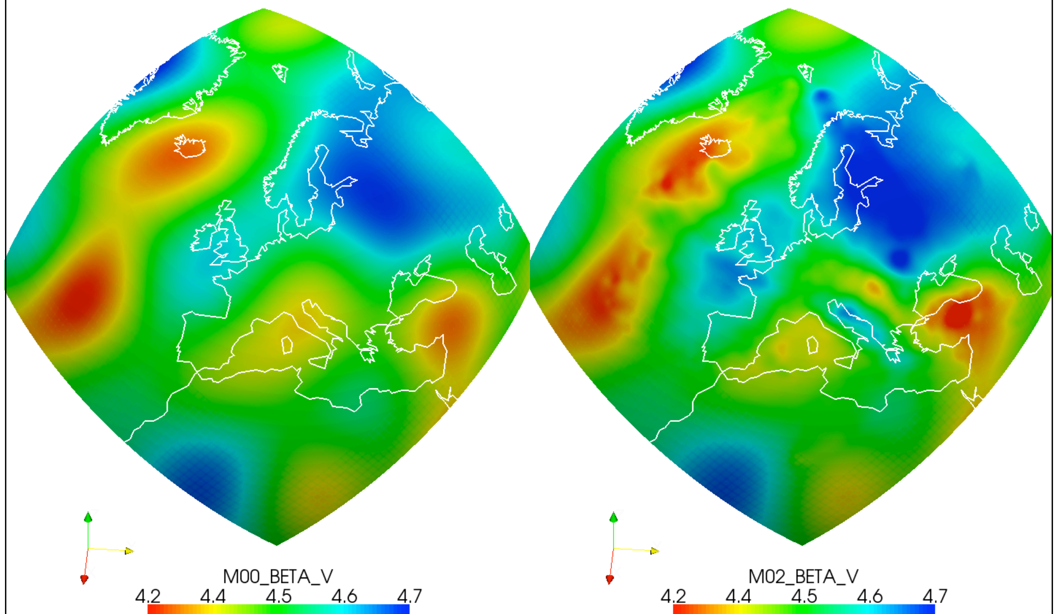

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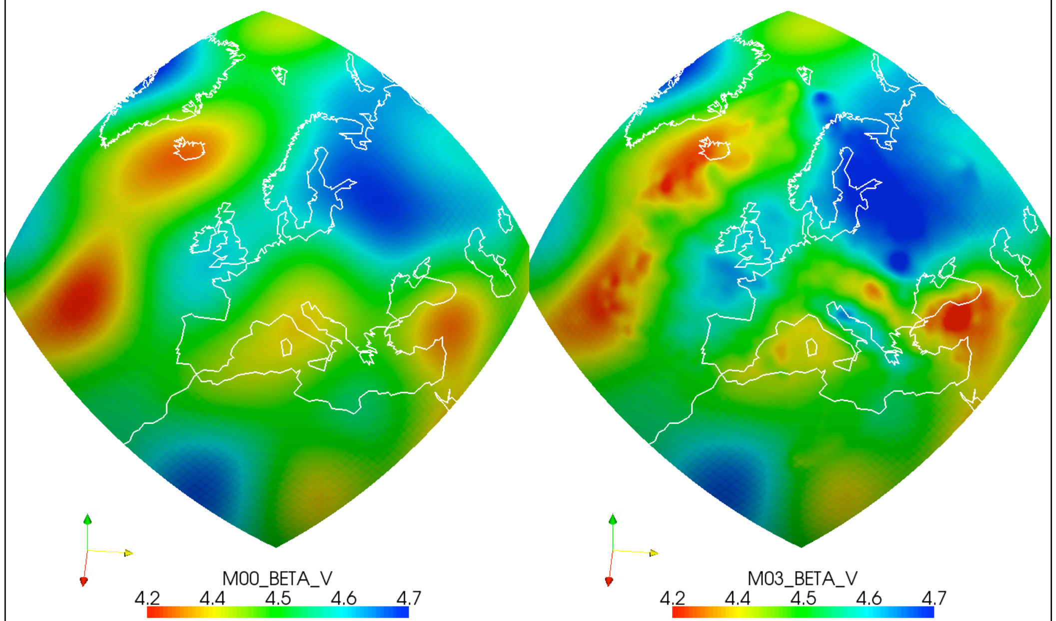

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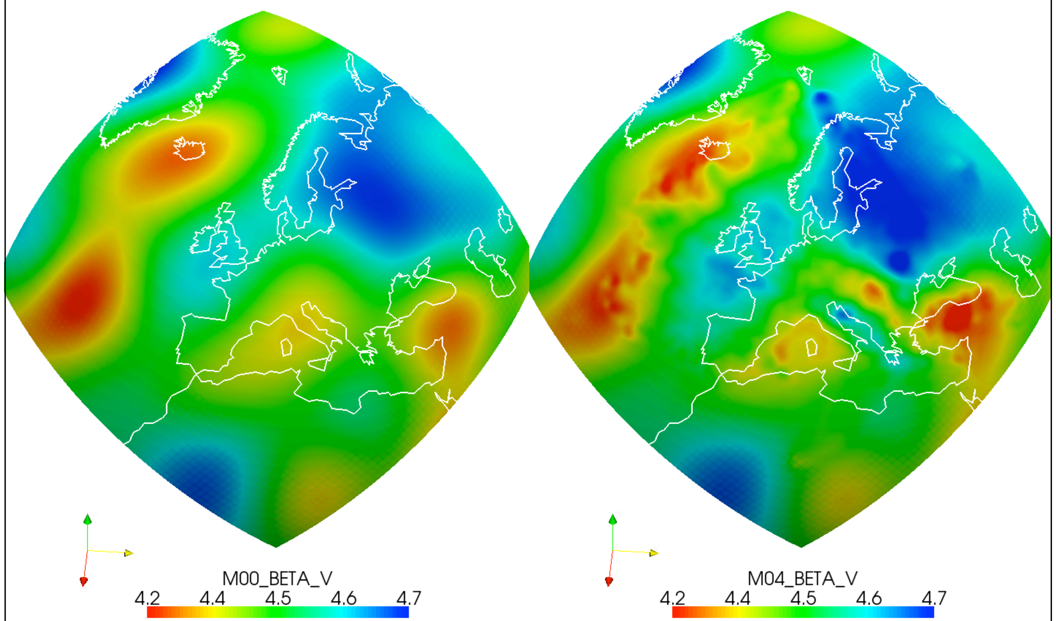

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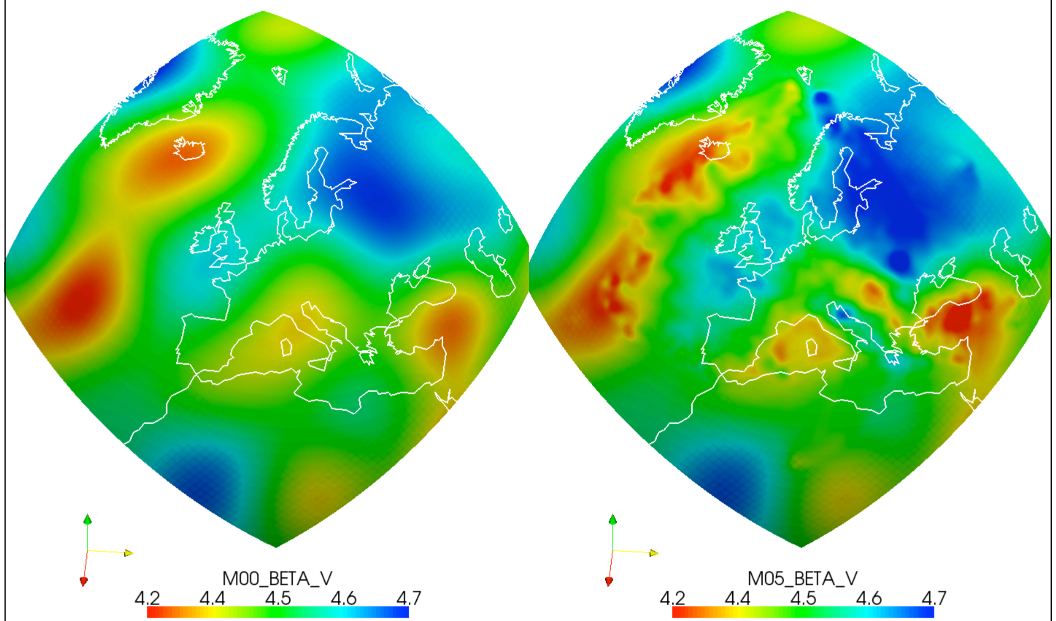

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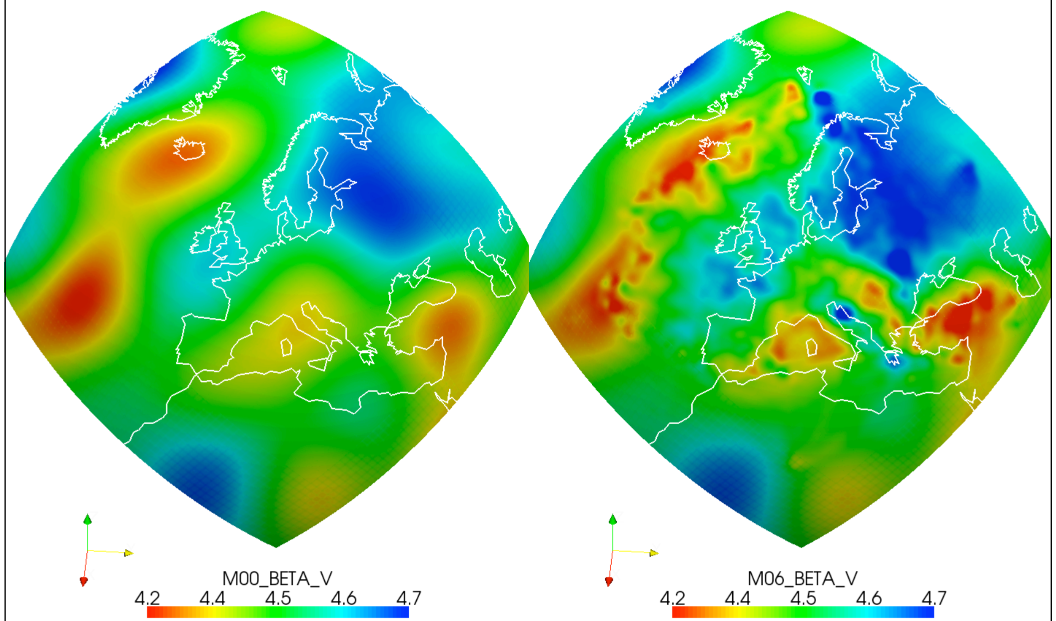

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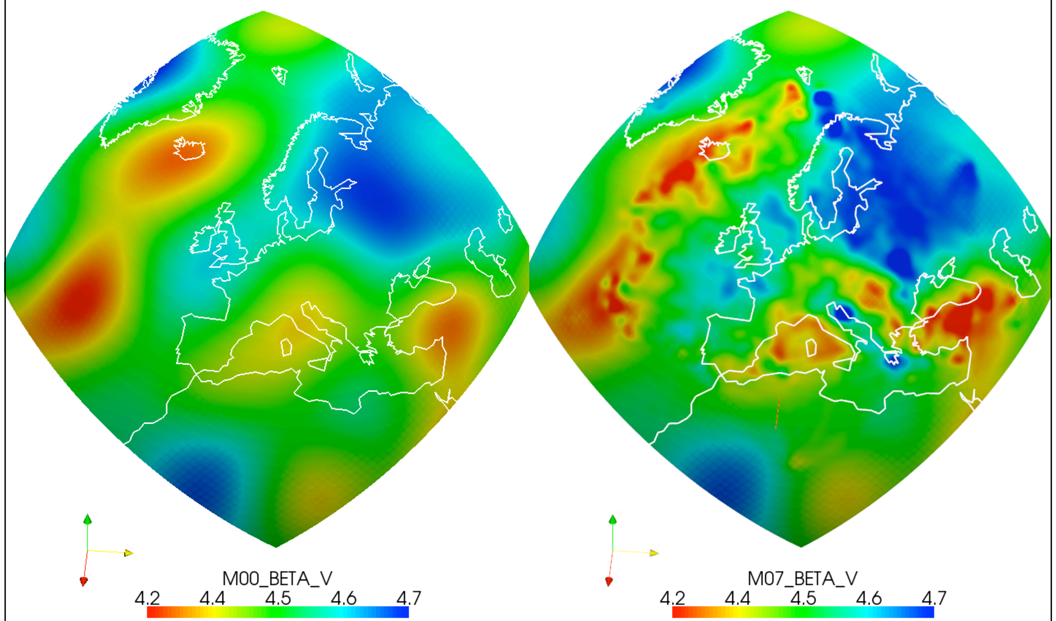

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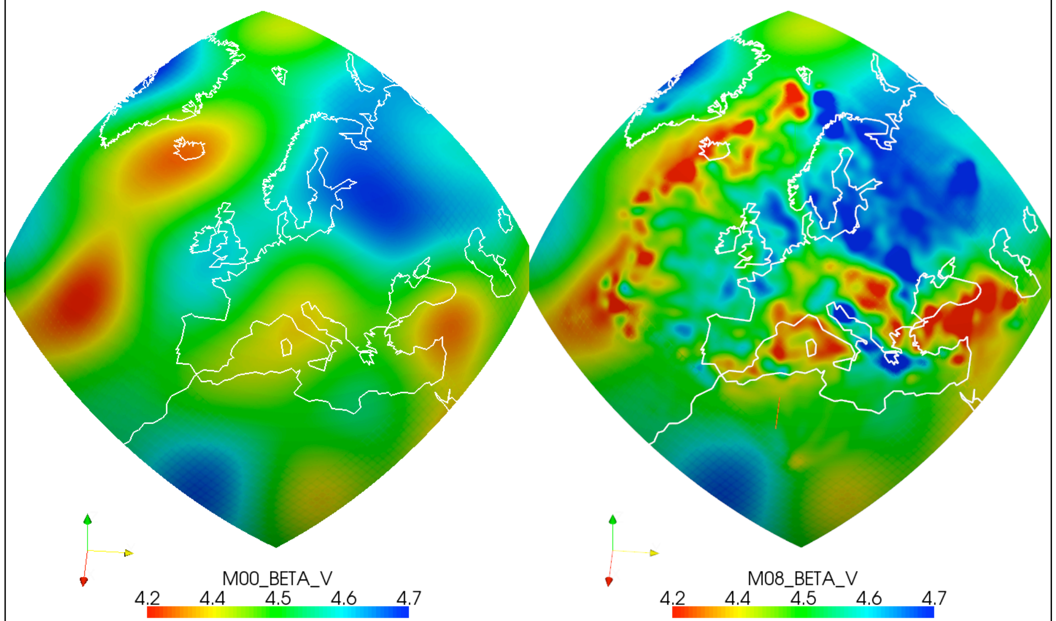

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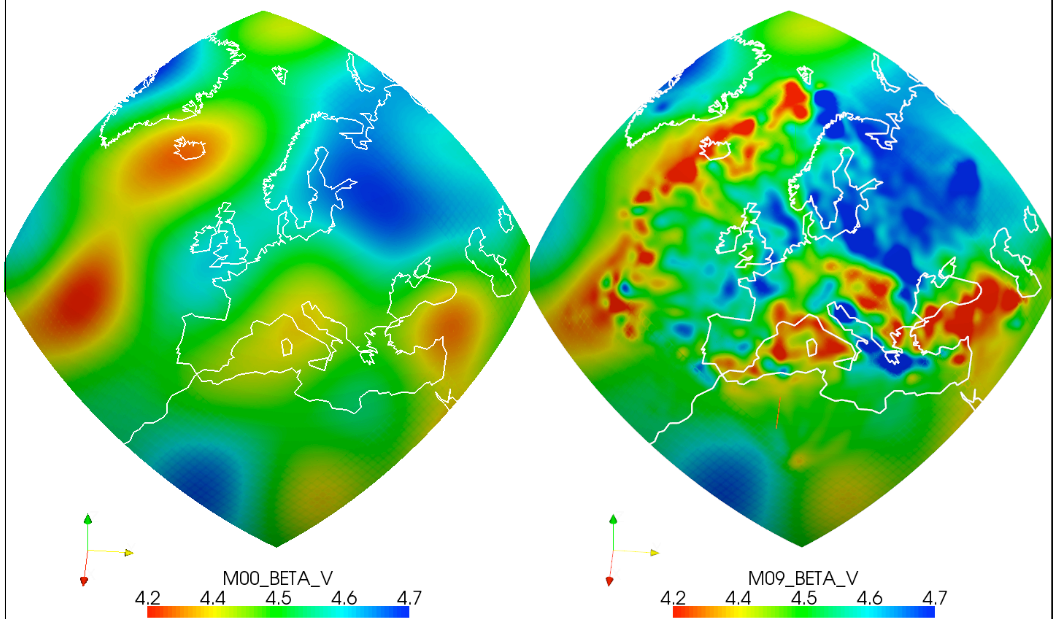

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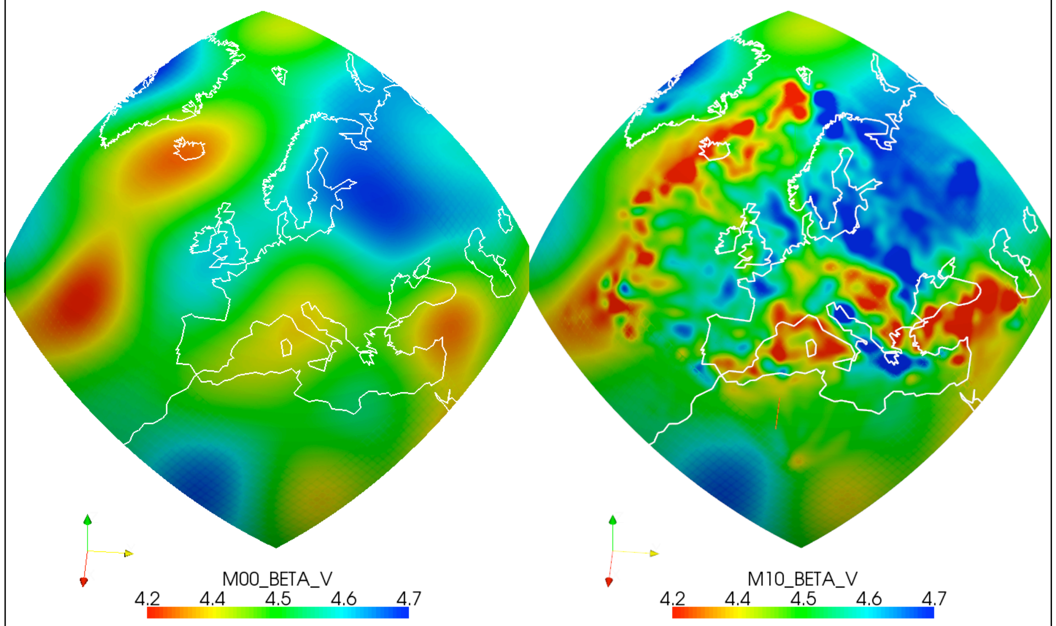

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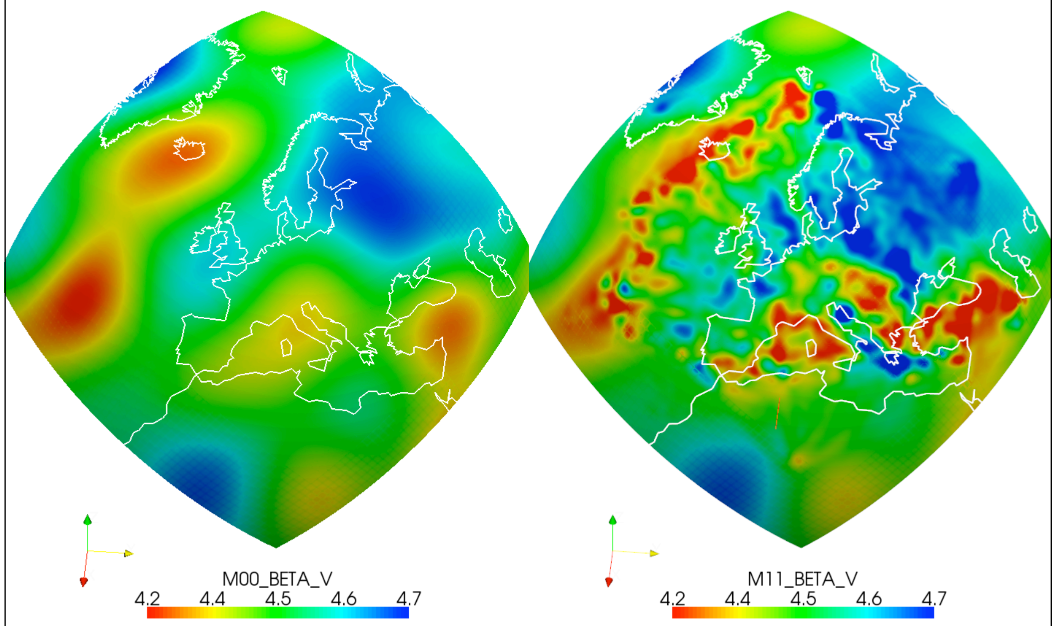

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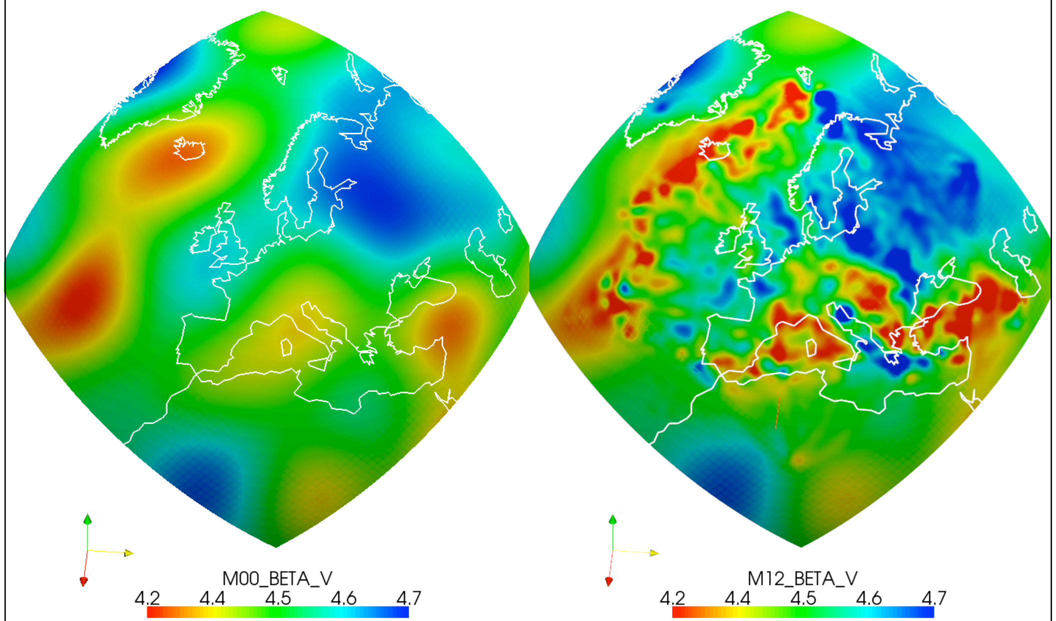

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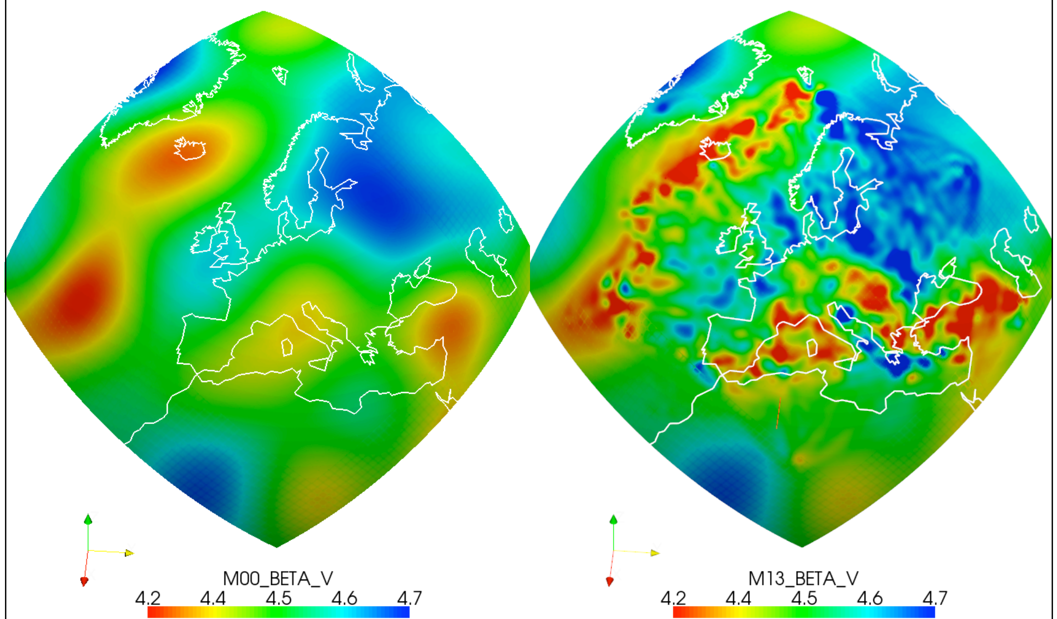

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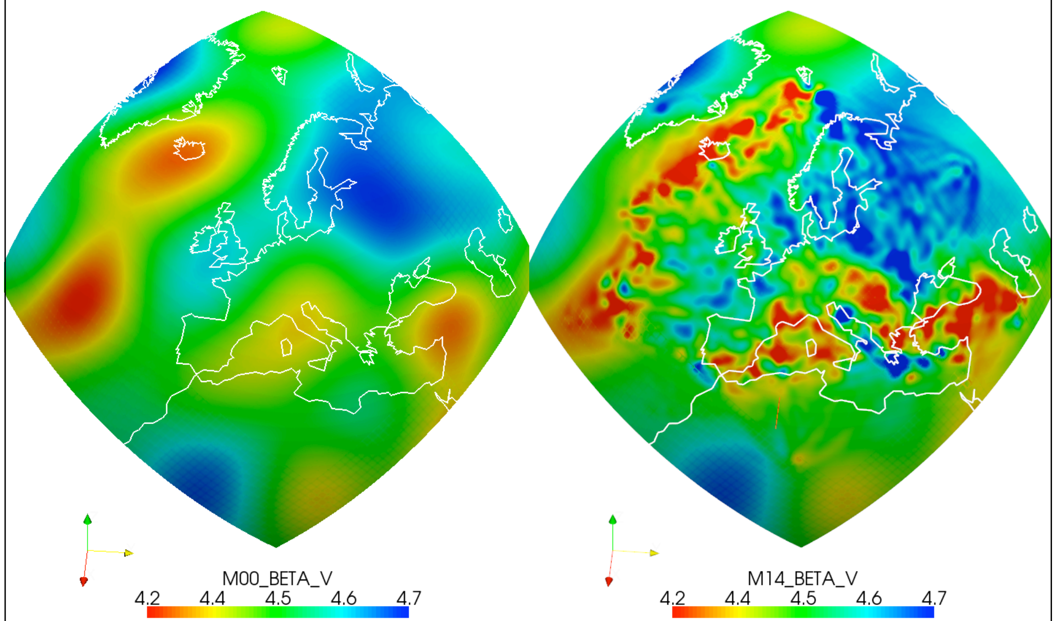

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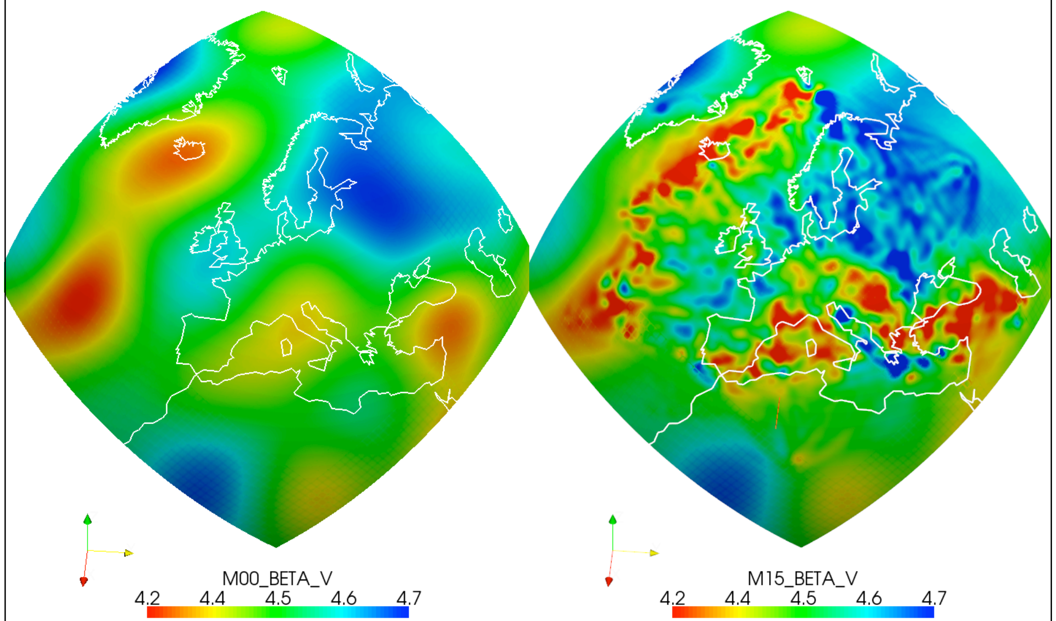

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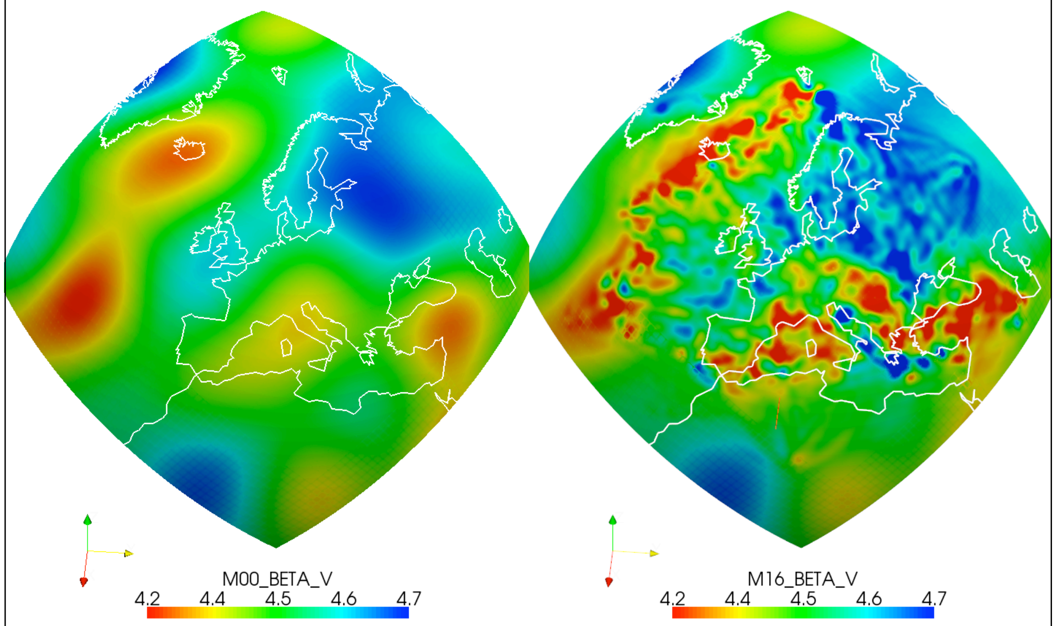

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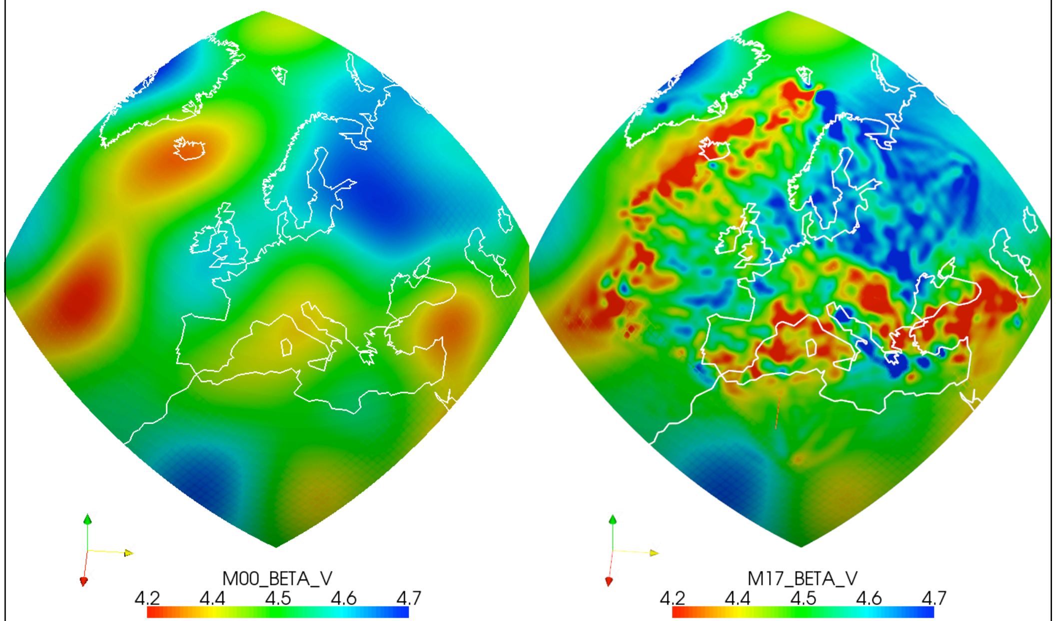

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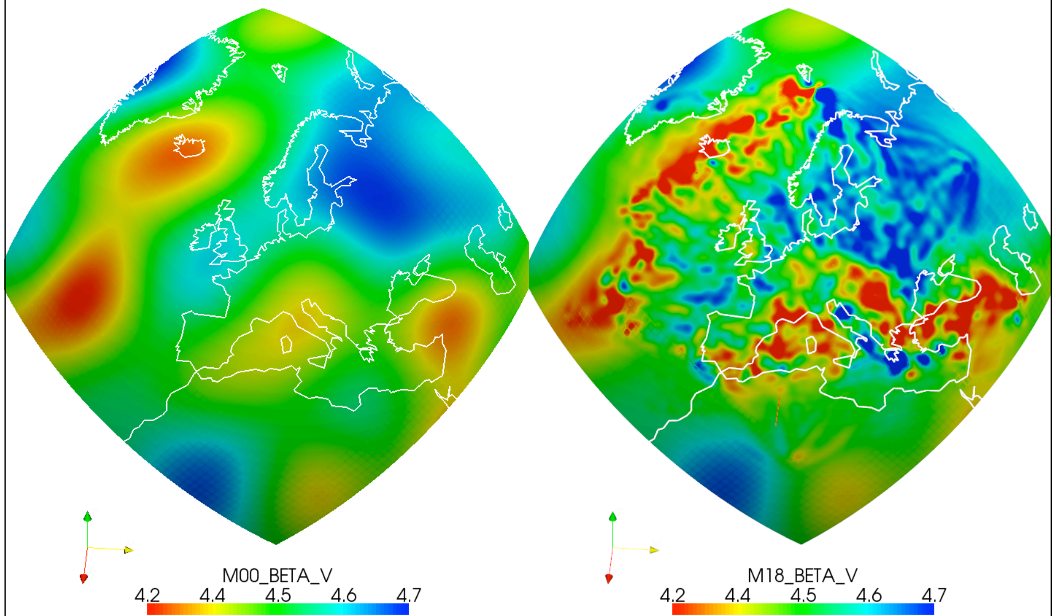

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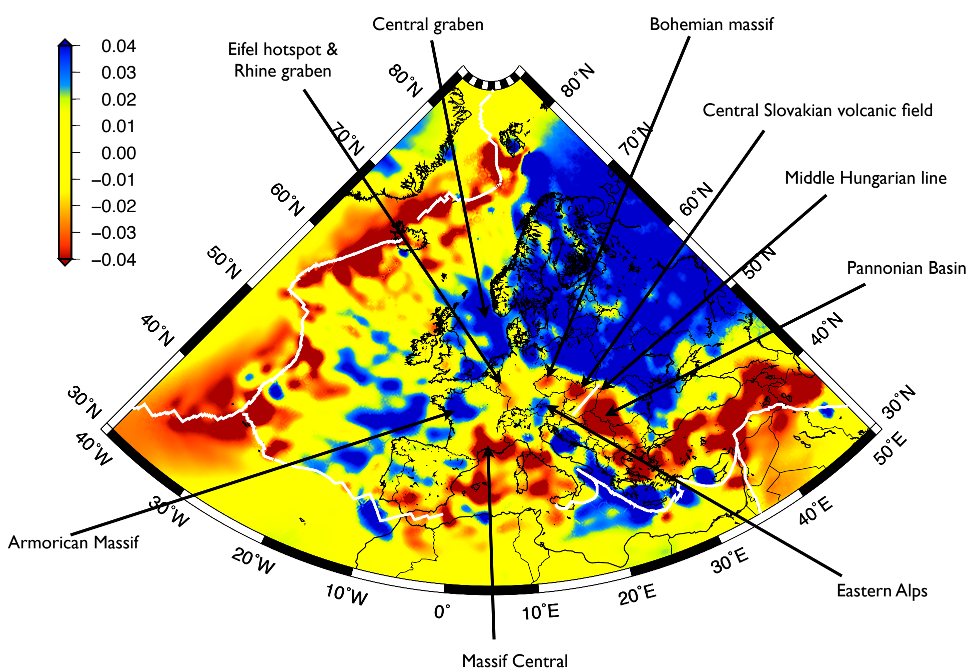
0.04
0.03
0.02
0.01
0.00
−0.01
−0.02
−0.03
−0.04
Central graben
Bohemian massif
Eifel hotspot &
Rhine graben
Central Slovakian volcanic field
Middle Hungarian line
Pannonian Basin
Armorican Massif
Eastern Alps
Massif Central
80°N
70°N
60°N
50°N
40°N
30°N
40°W
30°W
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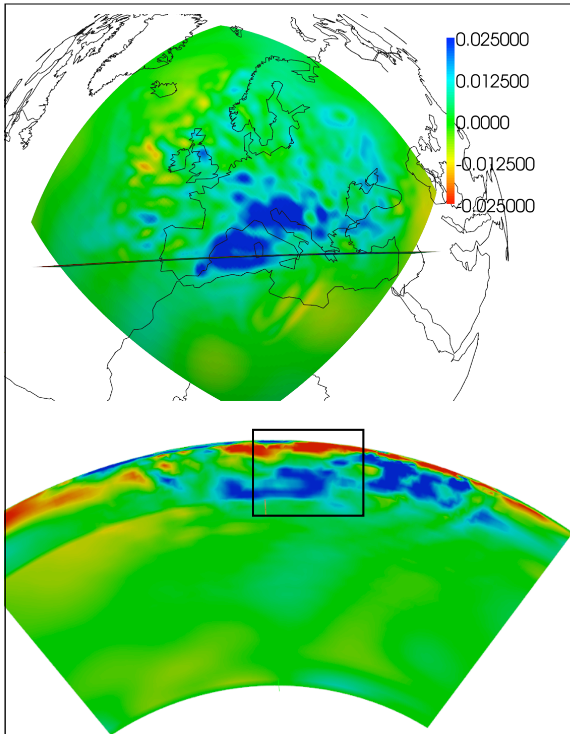
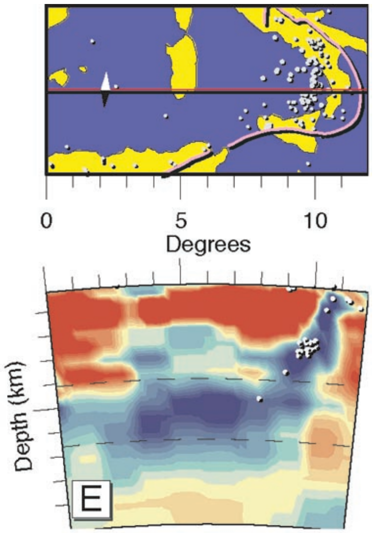
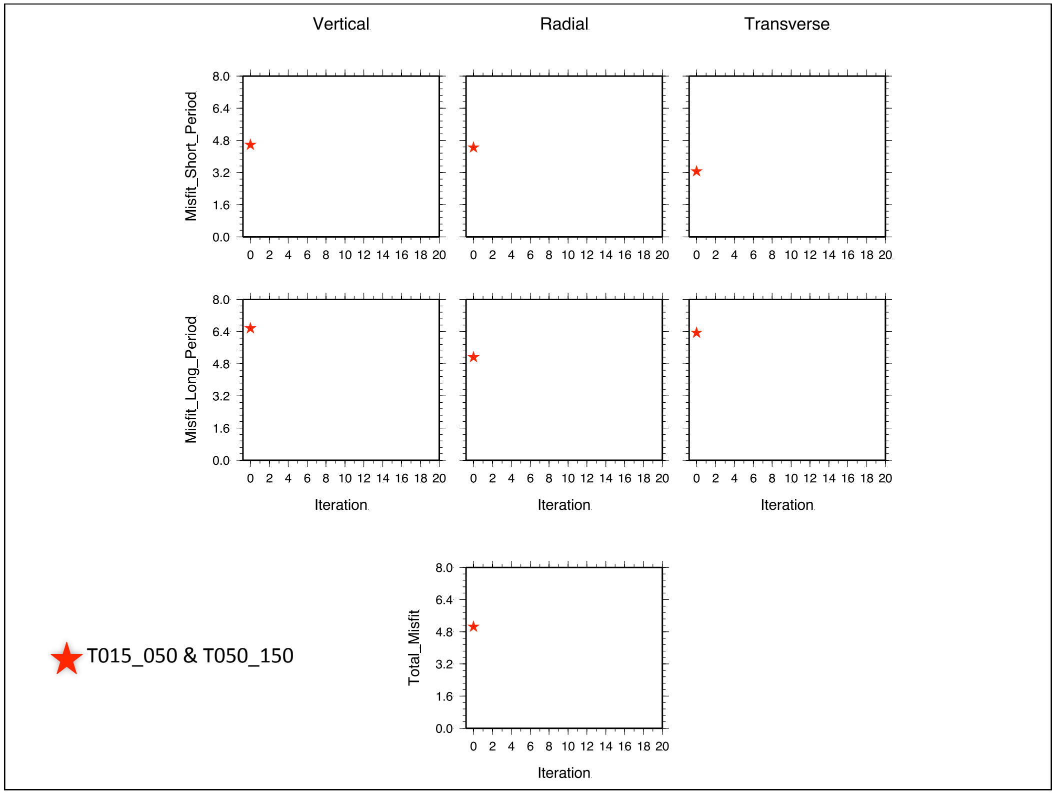
10°W
0°
10°E
20°E
30°E
40°E
50°E

Calabria

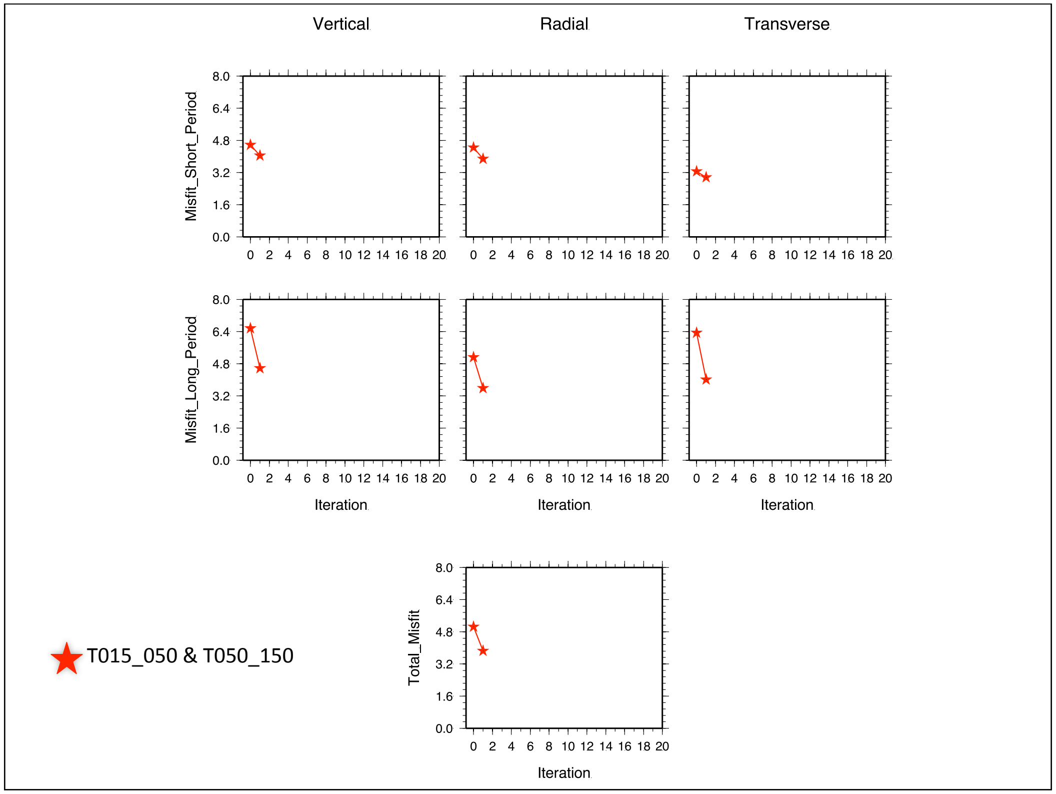




Wortel & Spakman (2000)



Vertical
Radial
Transverse
Misfit_Short_Period
Misfit_Long_Period
Total_Misfit
Iteration
T015_050 & T050_150



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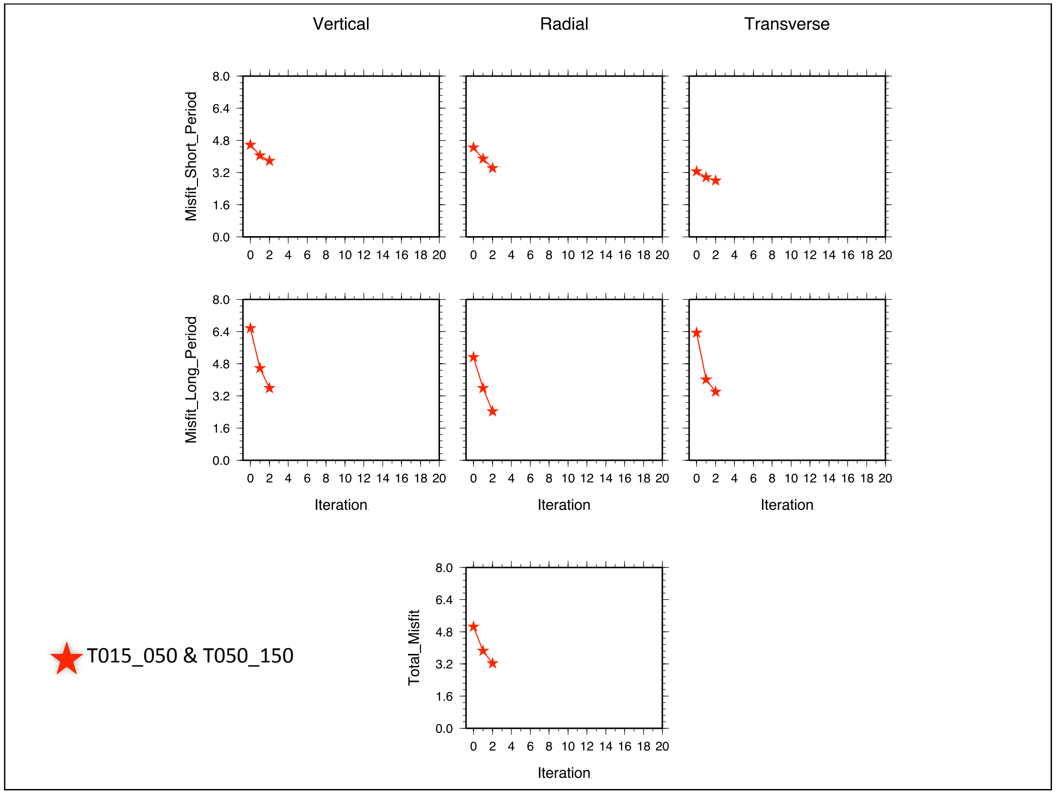
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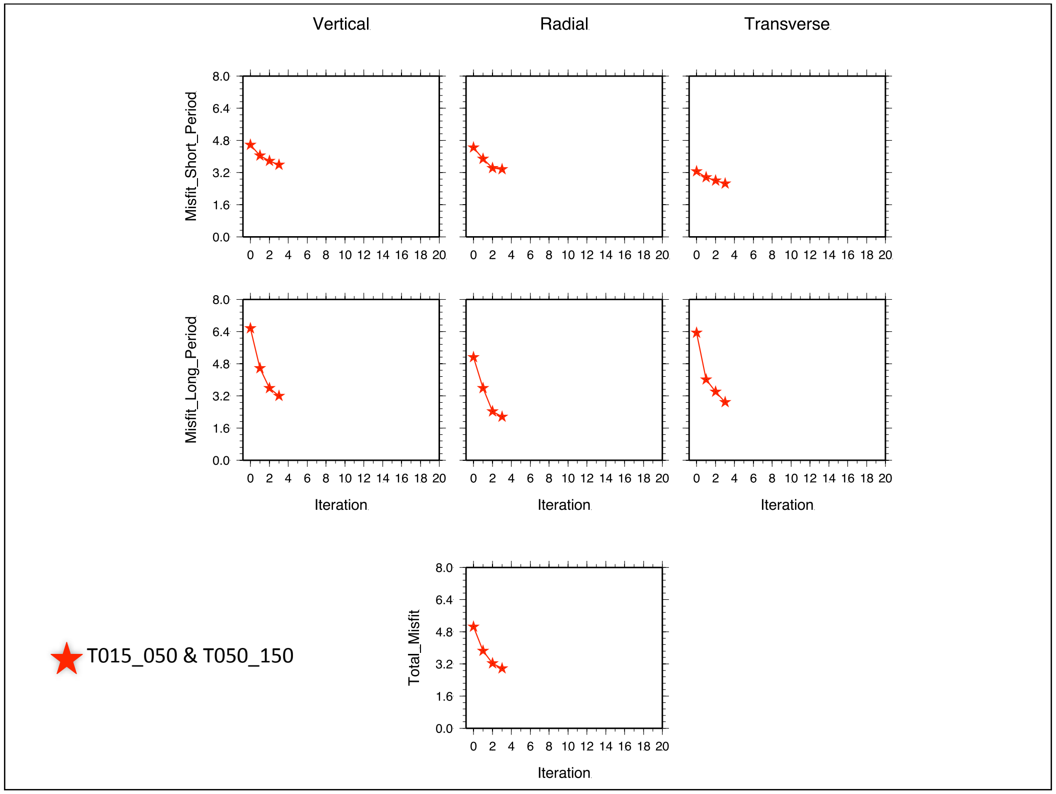
T015_050 & T050_150



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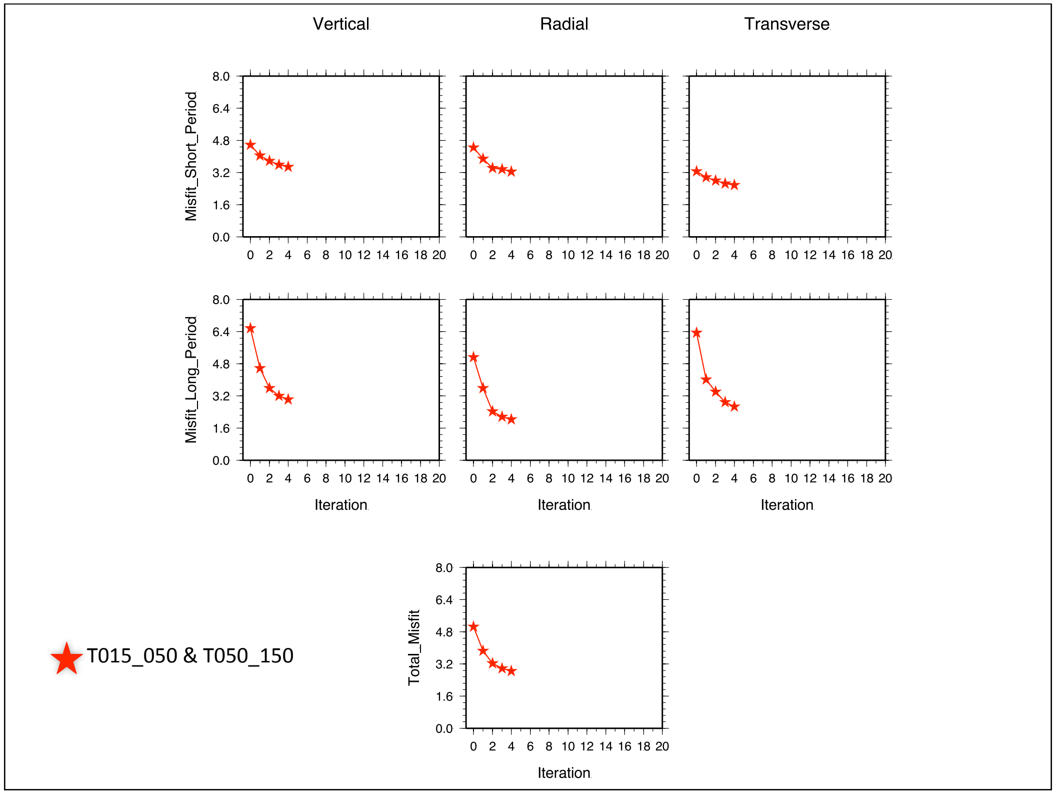
Iteration
Total_Misfit
T015_050 & T050_150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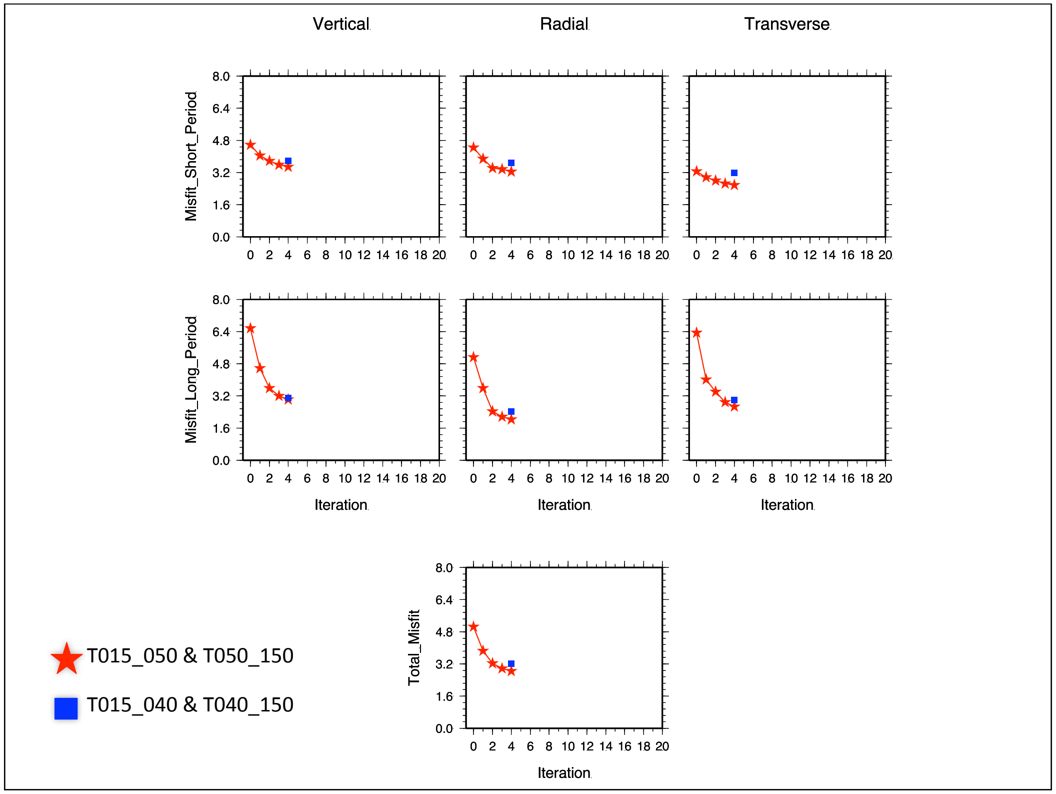

Vertical
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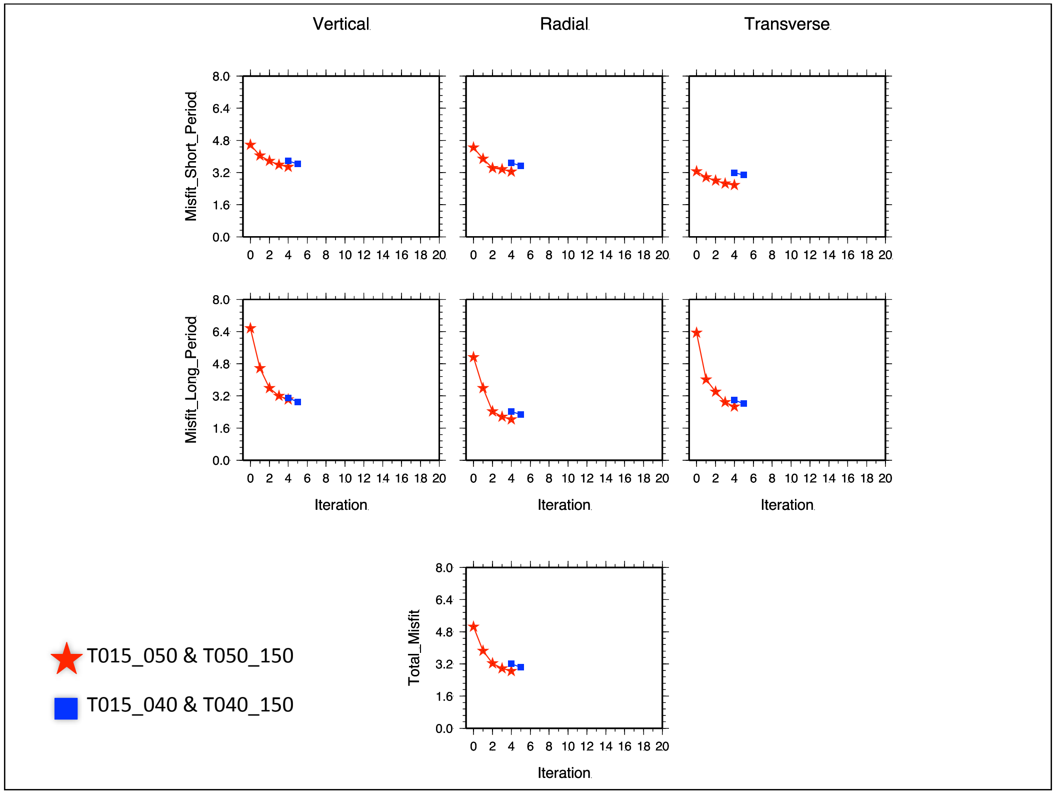
T015_050 & T050_150
T015_040 & T040_150



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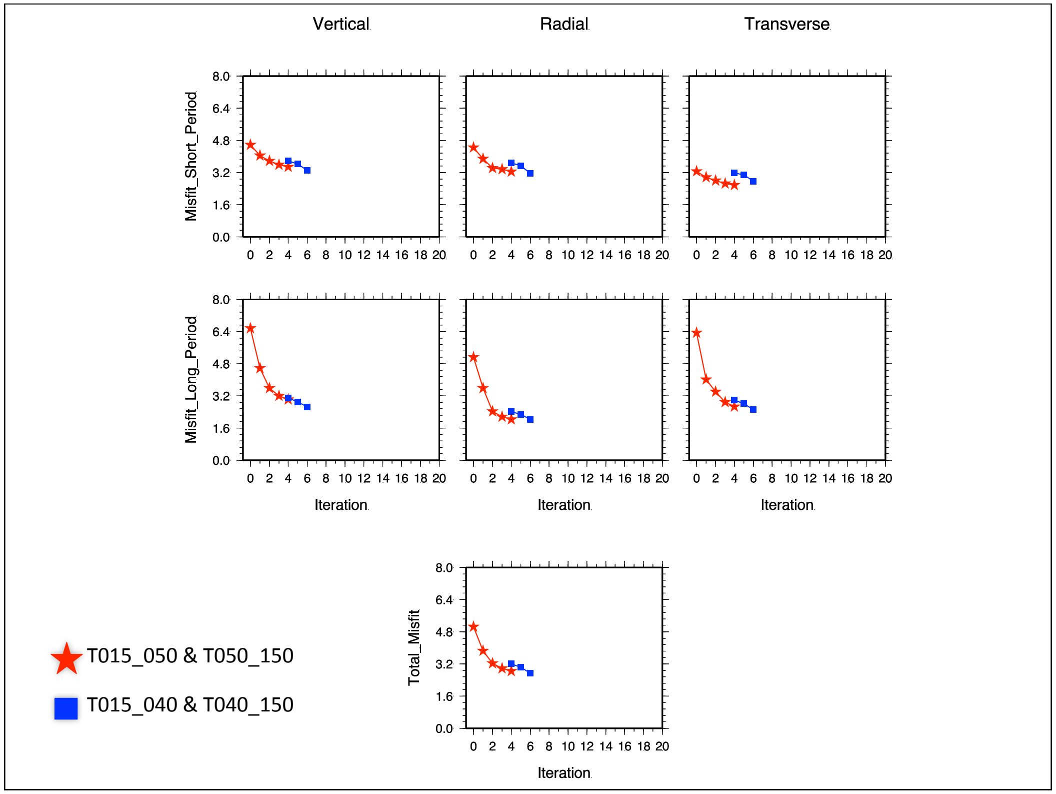
T015_040 & T040_150



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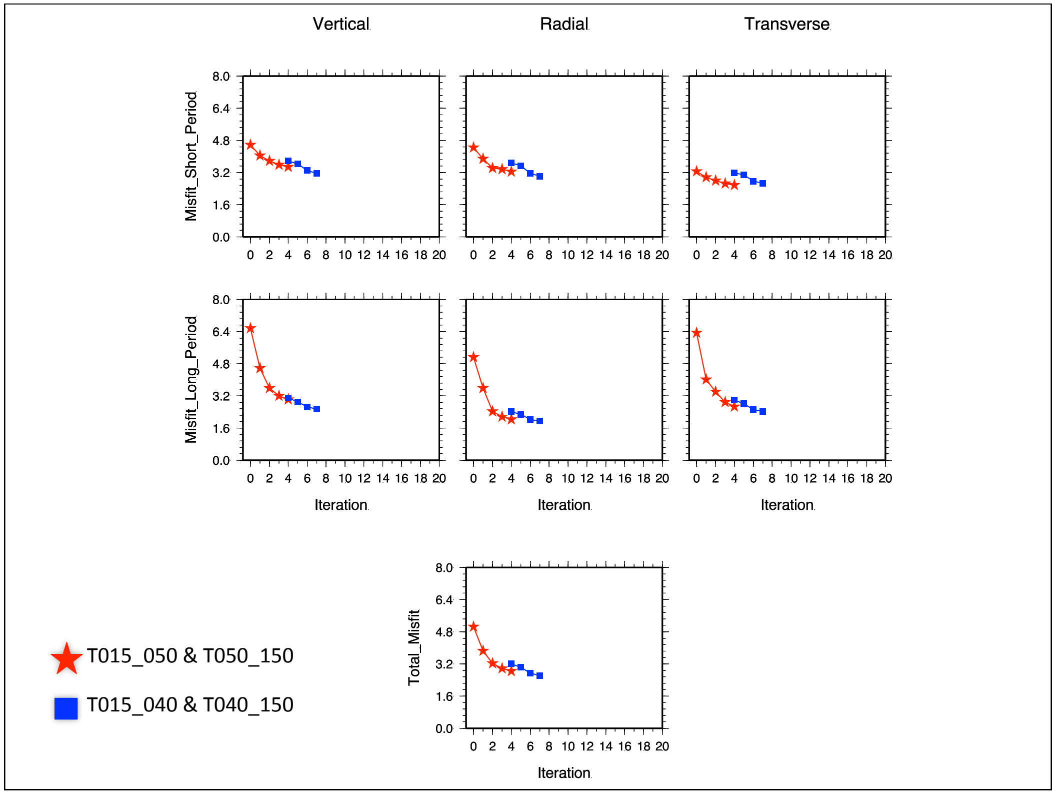
Iteration
T015_050 & T050_150
T015_040 & T040_150



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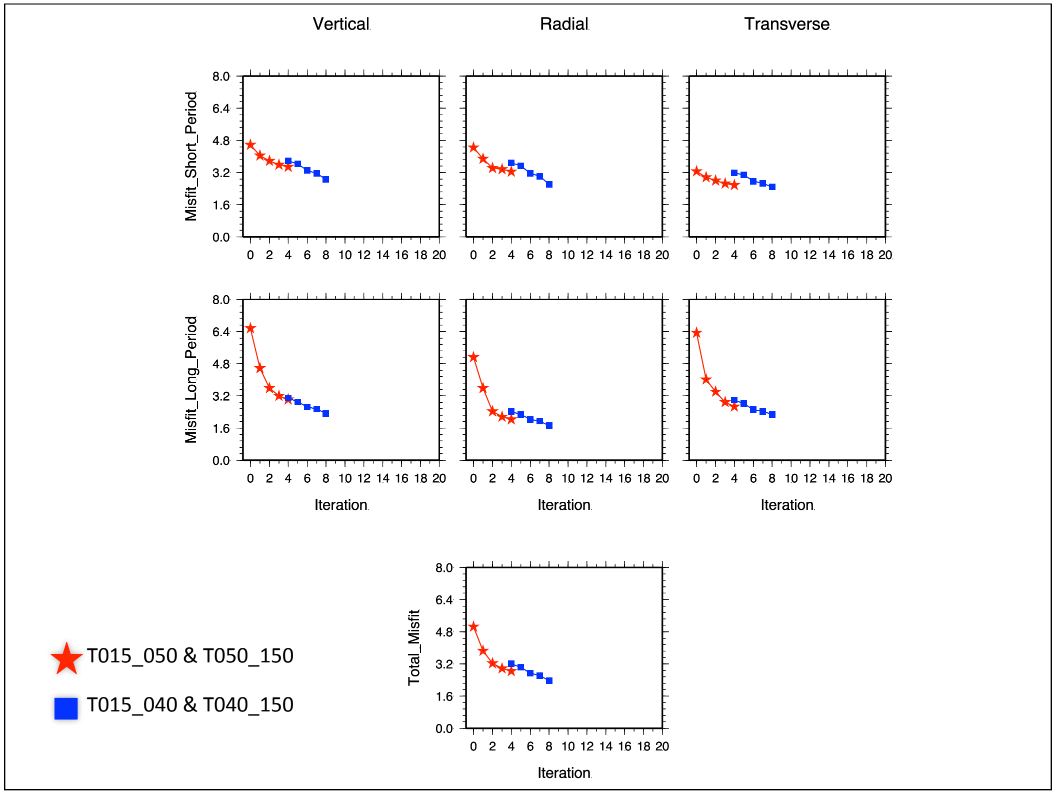

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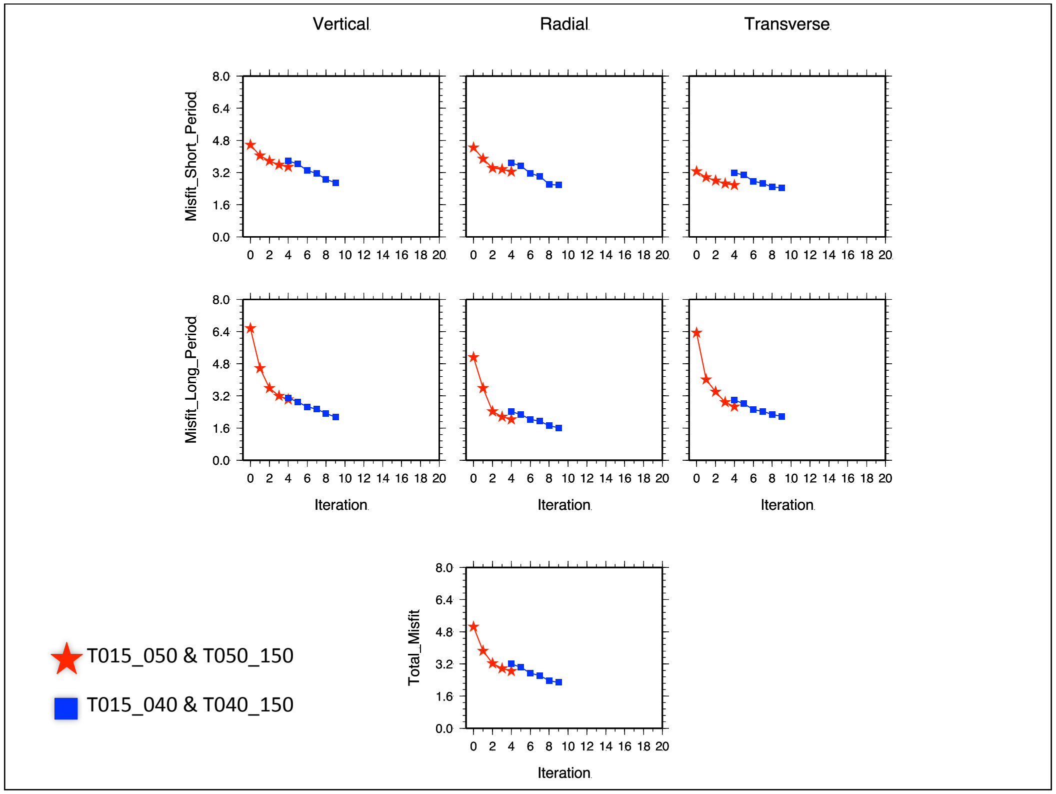
T015_050 & T050_150
T015_040 & T040_150



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Misfit_Short_Period
Misfit_Long_Period
Total_Misfit
Iteration
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4.8
3.2
1.6
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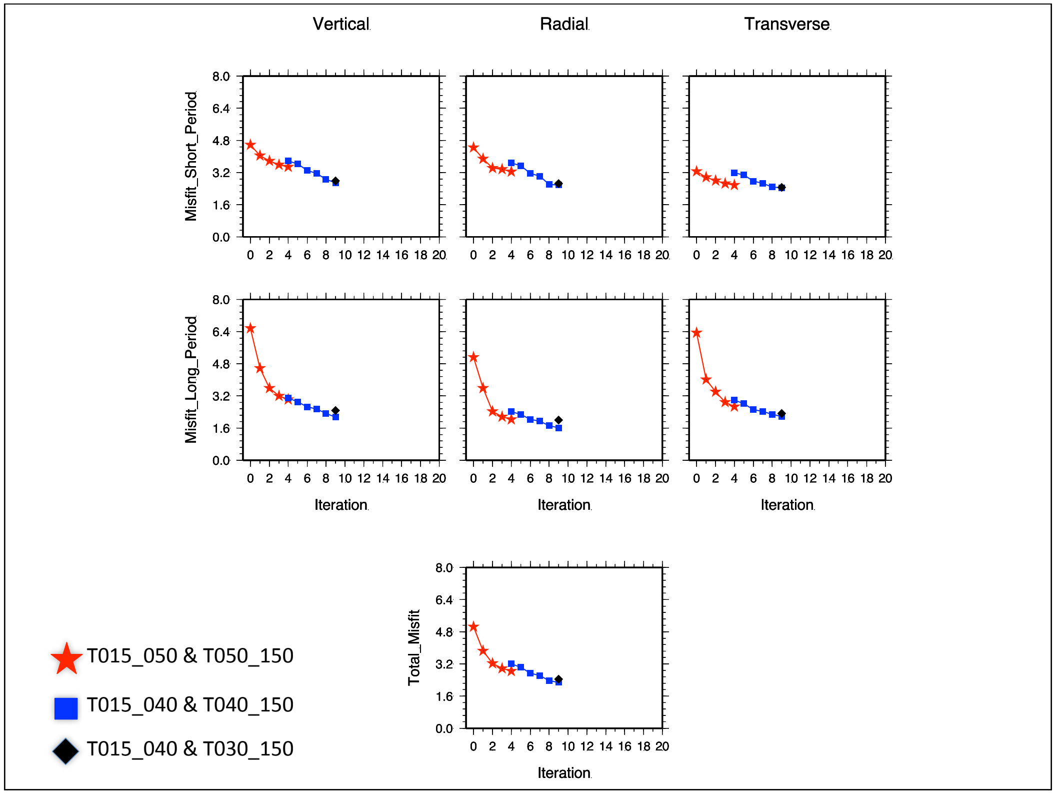
T015_050 & T050_150
T015_040 & T040_150



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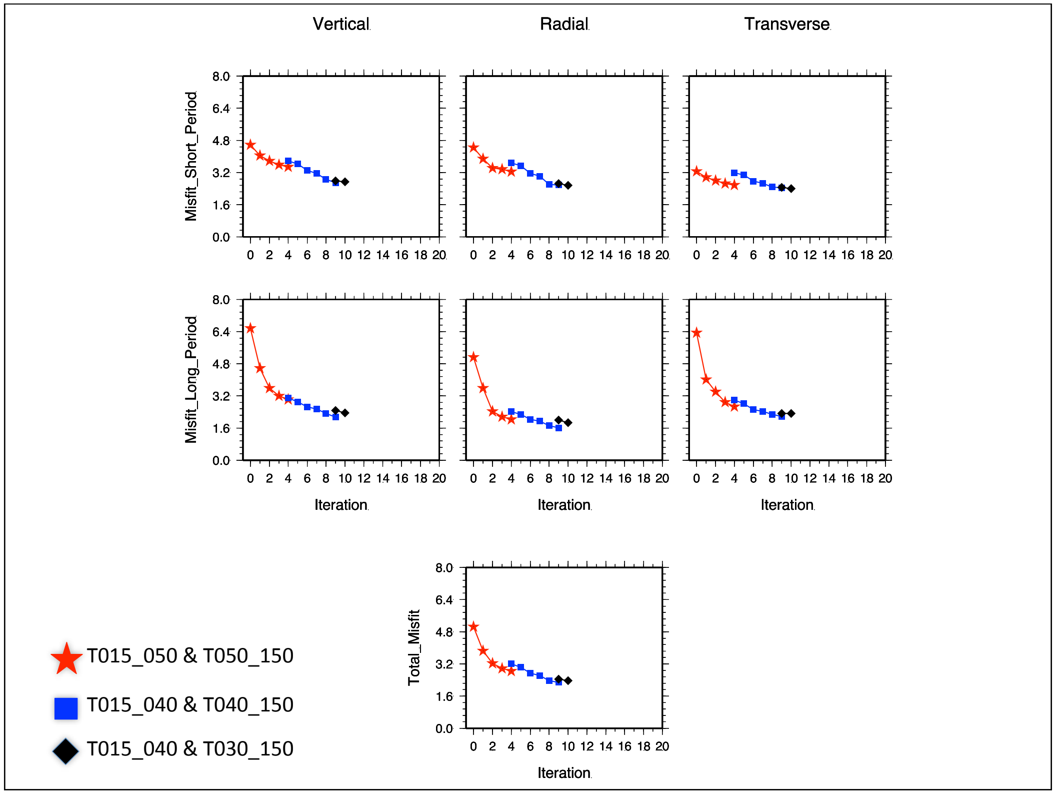
T015_040 & T030_150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Iteration
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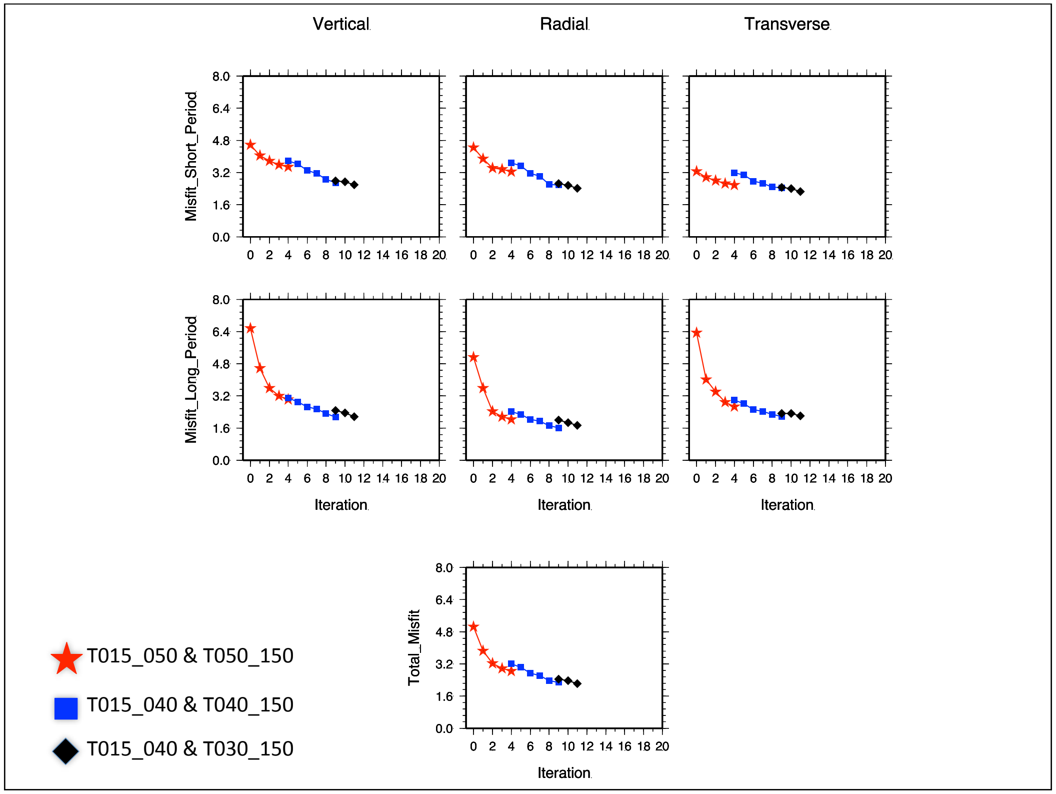

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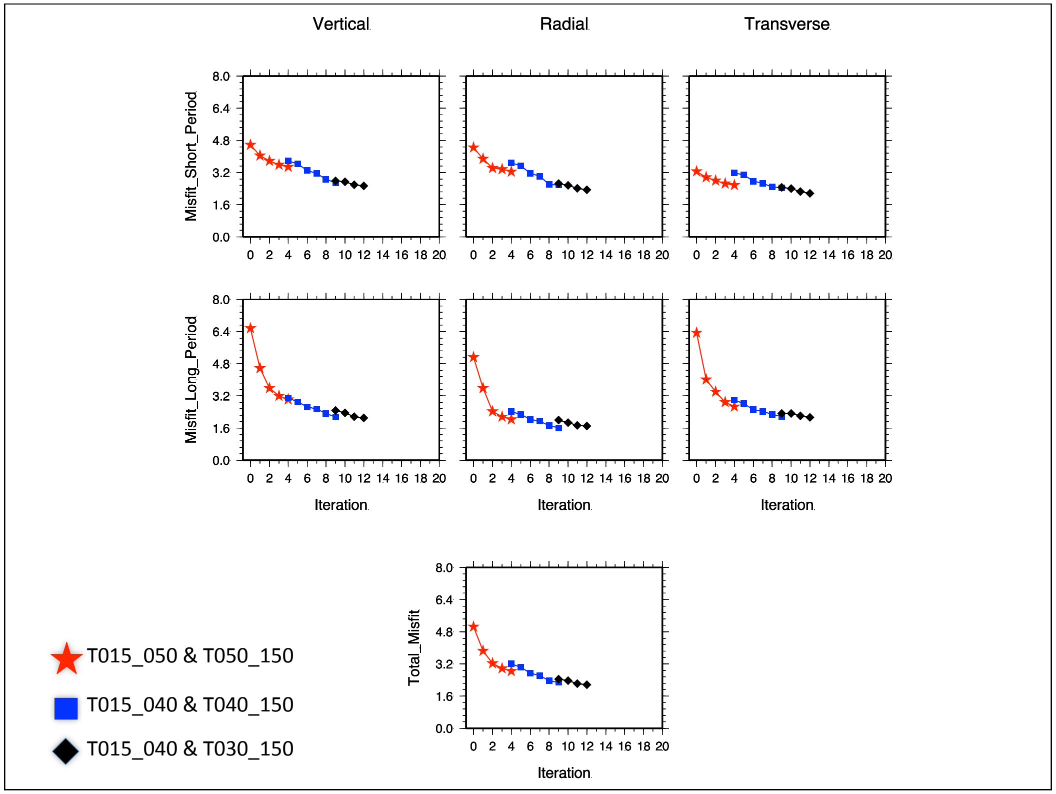

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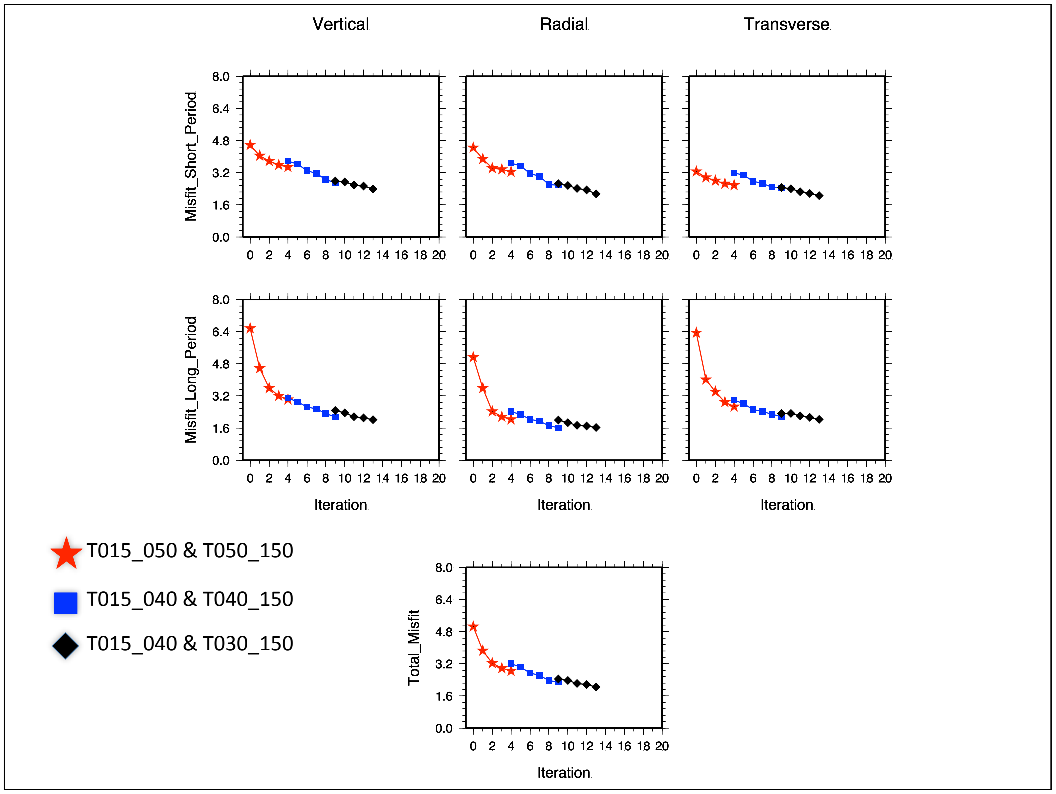

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T015_050 & T050_150
T015_040 & T040_150
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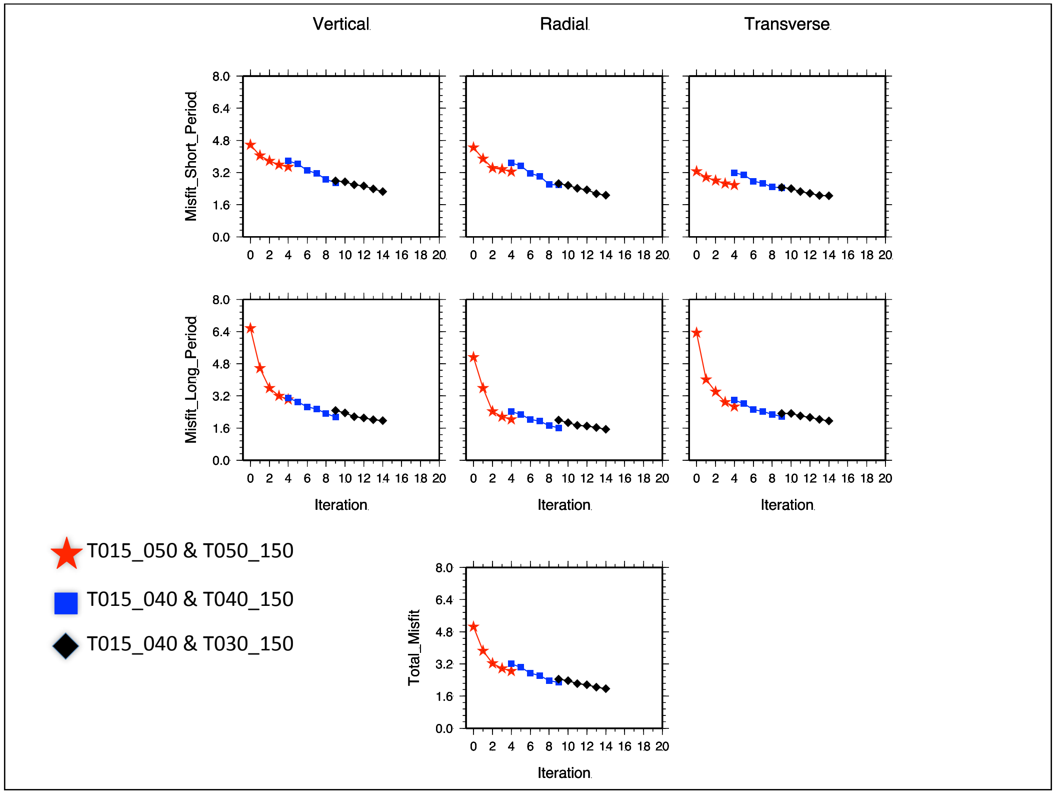

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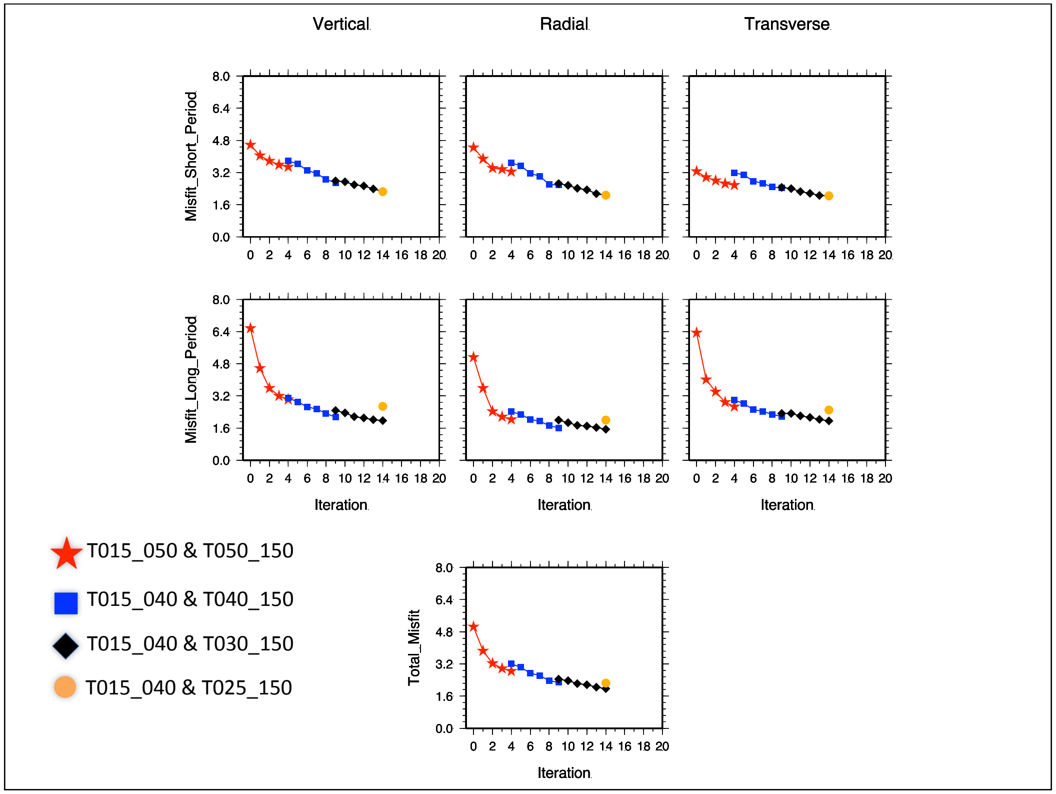
T015_040 & T040_150
T015_040 & T030_150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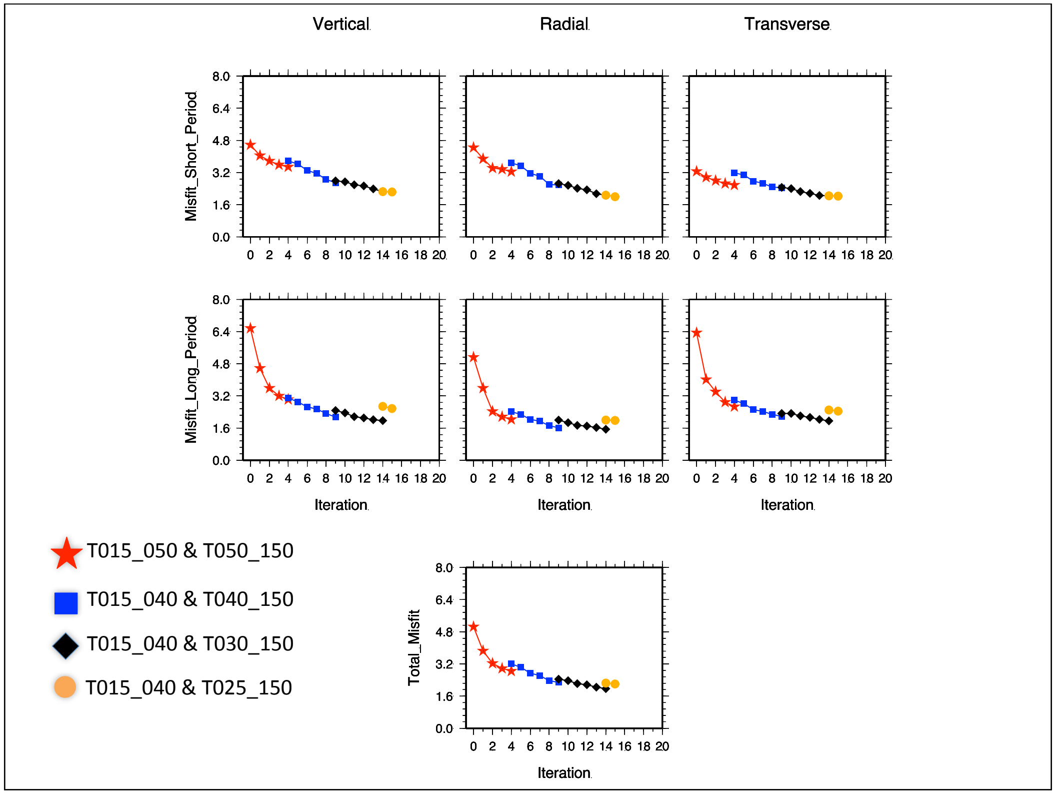
T015_040 & T025_150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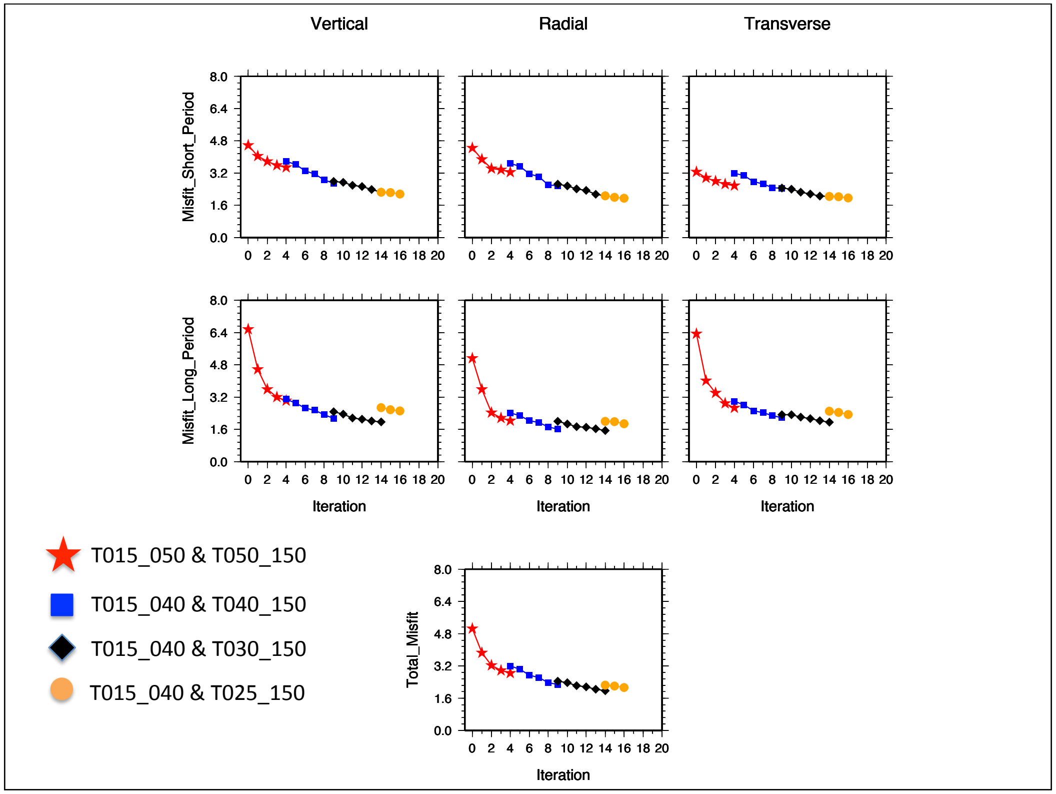
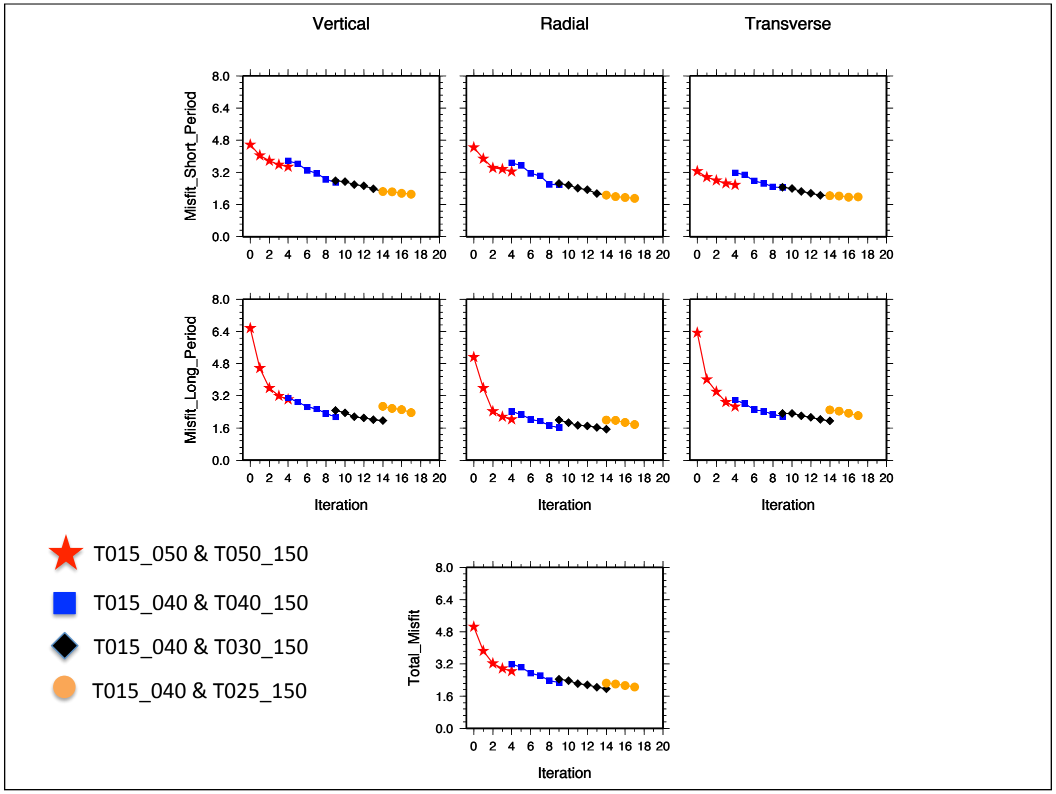
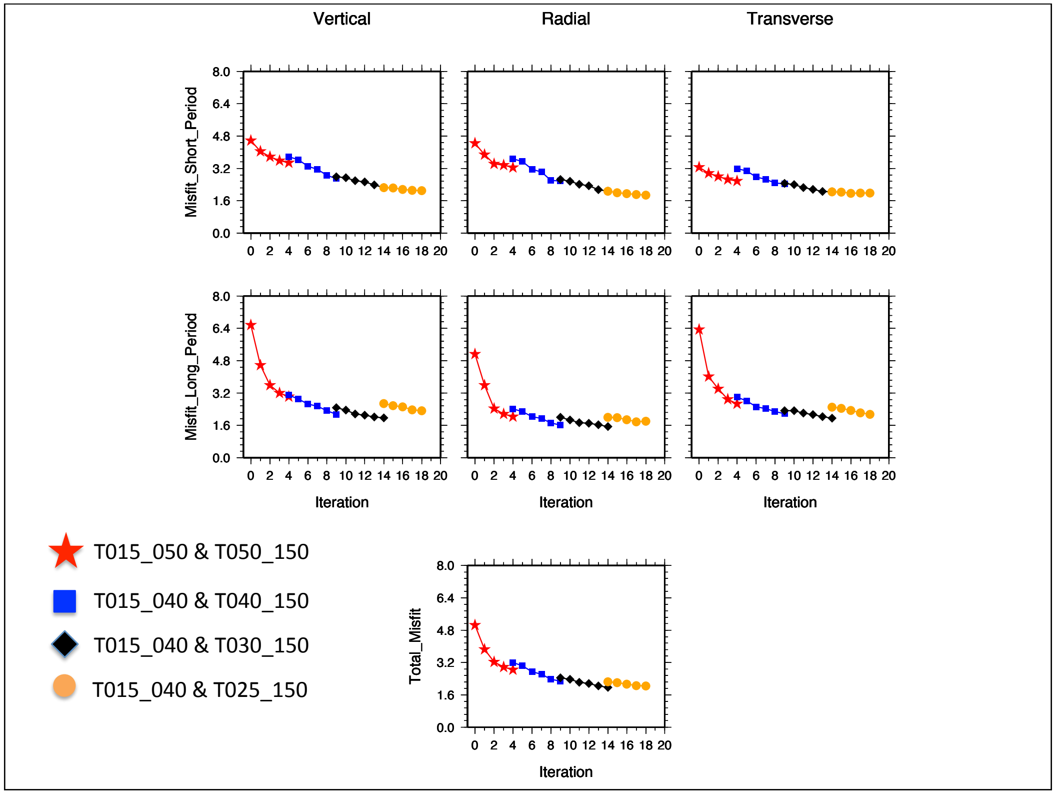
Total_Misfit
Iteration
T015_050 & T050_150
T015_040 & T040_150
T015_040 & T030_150
T015_040 & T025_150


Vertical
Radial
Transverse
Misfit_Short_Period
Misfit_Long_Period
Total_Misfit
Iteration
T015_050 & T050_150
T015_040 & T040_150
T015_040 & T030_150
T015_040 & T025_150



Vertical
Radial
Transverse
Misfit_Short_Period
Misfit_Long_Period
Total_Misfit
Iteration
T015_050 & T050_150
T015_040 & T040_150
T015_040 & T030_150
T015_040 & T025_150


Vertical
Radial
Transverse
Misfit_Short_Period
Misfit_Long_Period
Total_Misfit
Iteration
T015_050 & T050_150
T015_040 & T040_150
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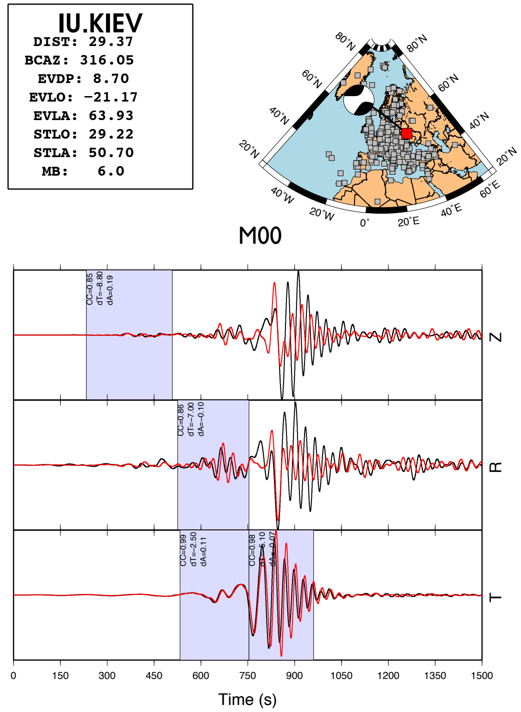
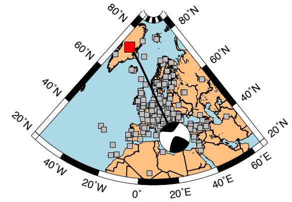
T015_040 & T025_150



20080529I546A

GE.SUMG

DIST: 45.37
BCAZ: 105.63
EVDP: 22.90
EVLO: 21.62
EVLA: 37.93
STLO: −38.45
STLA: 72.58
MB: 6.2



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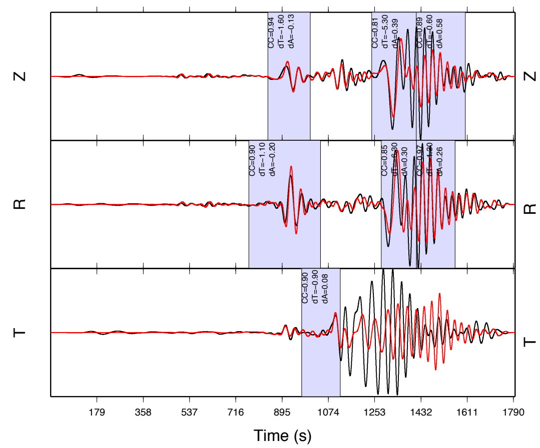

Conclusions

- **“Adjoint tomography”**
  - Goal: whole seismogram (frequency-dependent phase & amplitude)
  - Europe
  - Middle East
  - Entire globe!

	# earthquakes	# simulations	CPU core hours
Europe	160	11,200	806,400
Globe (Phase 1)	250	17,500	14,437,400
Globe (Phase 2)	5,000	350,000	739,200,000

- **Future extensions:**
  - Include amplitudes
  - Shear attenuation
  - More general anisotropy
- **Mathematical/computational challenges:**
  - Nonlinear, iterative inverse problems
  - Local timestepping, h-p adaptivity
  - GPU computing and optimization