Seismic Imaging based on Spectral-element and Adjoint Methods

Jeroen Tromp
(CMG collaboration with Ingrid Daubechies)

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Forward Problem in Seismology

Equation of motion:
\[ \rho \partial_t^2 s - \nabla \cdot T = f \]

Constitutive relationship (may be anelastic):
\[ T = c : \varepsilon \]

Boundary condition:
\[ \hat{n} \cdot T = 0 \]

Initial conditions:
\[ s(x, 0) = 0, \quad \partial_t s(x, 0) = 0 \]

Earthquake source:
\[ f = -M \cdot \nabla \delta(x - x_s) S(t) \]
Spectral-Element Simulations

Open Source software: SPECFEM3D_SESAME

Dimitri Komatitsch, Daniel Peter
Adjoint Tomography

PDE-constrained waveform tomography:

\[
\chi = \frac{1}{2} \sum_r \int_0^T \left[ s(x_r, t) - d(x_r, t) \right]^2 dt - \int_0^T \int_\Omega \lambda \cdot (\rho \partial_t^2 s - \nabla \cdot T - f) \, d^3x \, dt
\]

Change in the waveform misfit function:

\[
\delta \chi = \int_0^T \int_\Omega \sum_r [s(x_r, t) - d(x_r, t)] \delta(x - x_r) \cdot \delta s(x, t) \, d^3x \, dt
\]

\[
- \int_0^T \int_\Omega (\delta \rho \lambda \cdot \partial_t^2 s + \nabla \lambda \cdot \delta c \cdot \nabla s - \lambda \cdot \delta f) \, d^3x \, dt - \int_0^T \int_\Omega [\rho \partial_t^2 \lambda - \nabla \cdot (c \cdot \nabla \lambda)] \cdot \delta s \, d^3x \, dt
\]

\[
- \int_\Omega [\rho (\lambda \cdot \partial_t s - \partial_t \lambda \cdot \delta s)] \cdot \delta s \, d^3x - \int_0^T \int_{\partial \Omega} \mathbf{n} \cdot (c \cdot \nabla \lambda) \cdot \delta s \, d^3x \, dt.
\]
Adjoint Equations

Adjoint wavefield: \[ s^\dagger(x, t) \equiv \lambda(x, T - t) \]

Adjoint equation of motion: \[ \rho \partial^2_t s^\dagger = \nabla \cdot \mathbf{T}^\dagger + f^\dagger \]

Adjoint boundary conditions: \[ \hat{n} \cdot \mathbf{T}^\dagger = 0 \]

Adjoint initial conditions: \[ s^\dagger(x, 0) = 0, \quad \partial_t s^\dagger(x, 0) = 0 \]

Adjoint source: \[ f^\dagger(x, t) = \sum_{r=1}^{N} [s(x_r, T - t) - d(x_r, T - t)] \delta(x - x_r) \]
Frechet derivative

The Frechet derivative may be expressed as:

$$\delta \chi = \int_{\Omega} (\delta \rho K_\rho + \delta c : K_c) \, d^3 x + \int_0^T \int_{\Omega} s^\dagger \cdot \delta f \, d^3 x \, dt$$

Density and elastic tensor kernels:

$$K_\rho(x) = -\int_0^T s^\dagger(x, T - t) \cdot \partial_t^2 s(x, t) \, dt$$

$$K_c(x) = -\int_0^T \nabla s^\dagger(x, T - t) \nabla s(x, t) \, dt$$
Current & Future Research

- Model parameterization (transverse isotropy, Q, more general anisotropy)
- Misfit
  - Measurement type (cross-correlation traveltime, frequency-dependent phase & amplitude)
  - Window selection (steadily increase number of “picks”)
  - Band pass filtering (steadily increase frequency content)
- Model basis
- Regularization/smoothing
- Preconditioning
- Conjugate-gradient algorithm
- Sampling the posterior model distribution
- Resolution analysis (Hessian kernels)
- Validation
PhD thesis of Hejun Zhu
Misfit Function

\[ \chi(m) = \frac{1}{6} \sum_{x,z,t} \frac{1}{N_s} \sum_{N_p} \frac{1}{N_s} \sum_{N_p} \left( \frac{\Delta T_i}{\sigma_i} \right)^2 + \frac{1}{6} \sum_{x,z,t} \frac{1}{N_s} \sum_{N_p} \frac{1}{N_s} \sum_{N_p} \int \frac{W_i}{\sigma_i} \left( \frac{\Delta \tau(\omega)}{\sigma(\omega)} \right)^2 d\omega \]

- **Body-wave cross correlation/multitaper**
  - 15-40 s band pass
  - (1) P-SV (Vertical)
  - (2) P-SV (Radial)
  - (3) SH (Transverse)

- **Surface-wave multitaper**
  - 25-150 s band pass
  - (4) Rayleigh (Vertical)
  - (5) Rayleigh (Radial)
  - (6) Love (Transverse)

Adjoint source = SP (body-wave measurement) + LP (surface-wave measurement)

\[ \delta \chi^T = \int K_c \delta \ln c + K_{\beta_v} \delta \ln \beta_v + K_{\beta_h} \delta \ln \beta_h + K_\eta \delta \ln \eta + K_\rho \delta \ln \rho \, dV \]
Reduction in Misfit: First Iteration

- 50 minutes per simulation (168 cores)
- 3 simulations per event per iteration
- 1 iteration ~1 day
- Source + structure inversion: ~1.5M CPU hours
Depth 75 km
Depth 75 km
Depth 75 km
Depth 75 km
Depth 75 km
Depth 75 km
Depth 75 km
Depth 75 km
Depth 75 km
Depth 75 km
Depth 75 km
Depth 75 km
Depth 75 km
 Depth 75 km

- Eifel hotspot & Rhine graben
- Bohemian massif
- Central Slovakian volcanic field
- Central graben
- Middle Hungarian line
- Pannonian Basin
- Massif Central
- Armorican Massif
- Eastern Alps
dent information against which the reconstructions are tested. The principal outcome of these studies was an important one: the basic aspects of the tectonic reconstructions with east-southeastward migrating convergent plate boundaries in the western Mediterranean (Fig. 1) and subduction underneath the Hellenic arc agreed, in terms of predicted versus imaged slab length, with the upper mantle structure. Therefore, they can be used as a basis for further investigations. If we combine the tectonic reconstructions with results from studies on the effect of trench migration (46–51), the peculiar flat-lying slab at the base of the upper mantle is also accounted for and, in fact, supports roll-back.

From structure via hypothesis to process. As indicated above, gaps in the structure of subducted slabs suggest that slab detachment has occurred in several areas. Slab detachment, as such, is not a new feature in lithospheric dynamics; early seismicity-based studies speculated on the existence of detached slabs (52). However, we added a new element to the concept, the lateral migration of slab detachment (31). We hypothesized that a small tear in the slab initiates lateral rupture propagation (Fig. 3). The physical basis for this process stems from the notion that the distribution of the slab pull is affected by a tear in the slab. In the segment of the plate boundary where the slab is detached, the slab pull is not transferred to the lithosphere at the surface. Instead, the weight of the slab is at least partially supported by the still continuous part of the slab (Fig. 3), thereby concentrating the slab pull force.

Stress concentration, with down-dip tension, near the tip of the tear causes further propagation. From the seismic tomography results, we determined three regions where the migrating slab detachment process may have occurred (or may still be occurring):

Fig. 2. Tomographic images of P-wave velocity anomalies (39) for the Mediterranean/Carpathian region. Colors indicate seismic wave speed anomalies as percentage deviations from average mantle velocities given by the one-dimensional reference model ak135 (113). (A) and (B) show map view images at 200 and 600 km depth, respectively. Projection and map dimensions are the same as in Fig. 1. Shadowed pink lines show the tectonic outlines similar to Fig. 1. Contouring scale ranges between –X% and X%, where X/11005 2.5 in (A), (C), and (D), and X/11005 1.5 in (B) and (E) through (J). (C and D) Blow-up for the Apennines-Calabria region at 53 and 380 km depth. (E through J) Vertical slices computed along great-circle segments (red line in map); above each slice, the map provides geographical orientation. The white arrow of the compass needle points north. The horizontal axis is in degrees along the great-circle segment defining the slice (straight red line in map). The vertical axis shows depth with tics at 100-km intervals. White dots indicate earthquakes. The dashed lines in the tomographic section indicate the 410 and 660 km discontinuities. (E) and (F) are sections through the Calabrian arc and southern Apenines. (G) and (H) are sections through the Carpathian-Pannonian region and (I) and (J) through the Aegean region.
T015_050

&

T050_150

Vertical
Radial
Transverse

8.0
6.4
4.8
3.2
1.6
0.0

0 2 4 6 8 10 12 14 16 18 20

Iteration

8.0
6.4
4.8
3.2
1.6
0.0

0 2 4 6 8 10 12 14 16 18 20

Iteration

8.0
6.4
4.8
3.2
1.6
0.0

0 2 4 6 8 10 12 14 16 18 20

Iteration

T015_050 & T050_150

Total Mean
Conclusions

• “Adjoint tomography”
  • Goal: whole seismogram (frequency-dependent phase & amplitude)
  • Europe
  • Middle East
  • Entire globe!

<table>
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<th></th>
<th># earthquakes</th>
<th># simulations</th>
<th>CPU core hours</th>
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<tbody>
<tr>
<td>Europe</td>
<td>160</td>
<td>11,200</td>
<td>806,400</td>
</tr>
<tr>
<td>Globe (Phase 1)</td>
<td>250</td>
<td>17,500</td>
<td>14,437,400</td>
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<tr>
<td>Globe (Phase 2)</td>
<td>5,000</td>
<td>350,000</td>
<td>739,200,000</td>
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• Future extensions:
  • Include amplitudes
  • Shear attenuation
  • More general anisotropy

• Mathematical/computational challenges:
  • Nonlinear, iterative inverse problems
  • Local timestepping, h-p adaptivity
  • GPU computing and optimization