A Statistical Generalization of the Transformed Eulerian Mean Circulation

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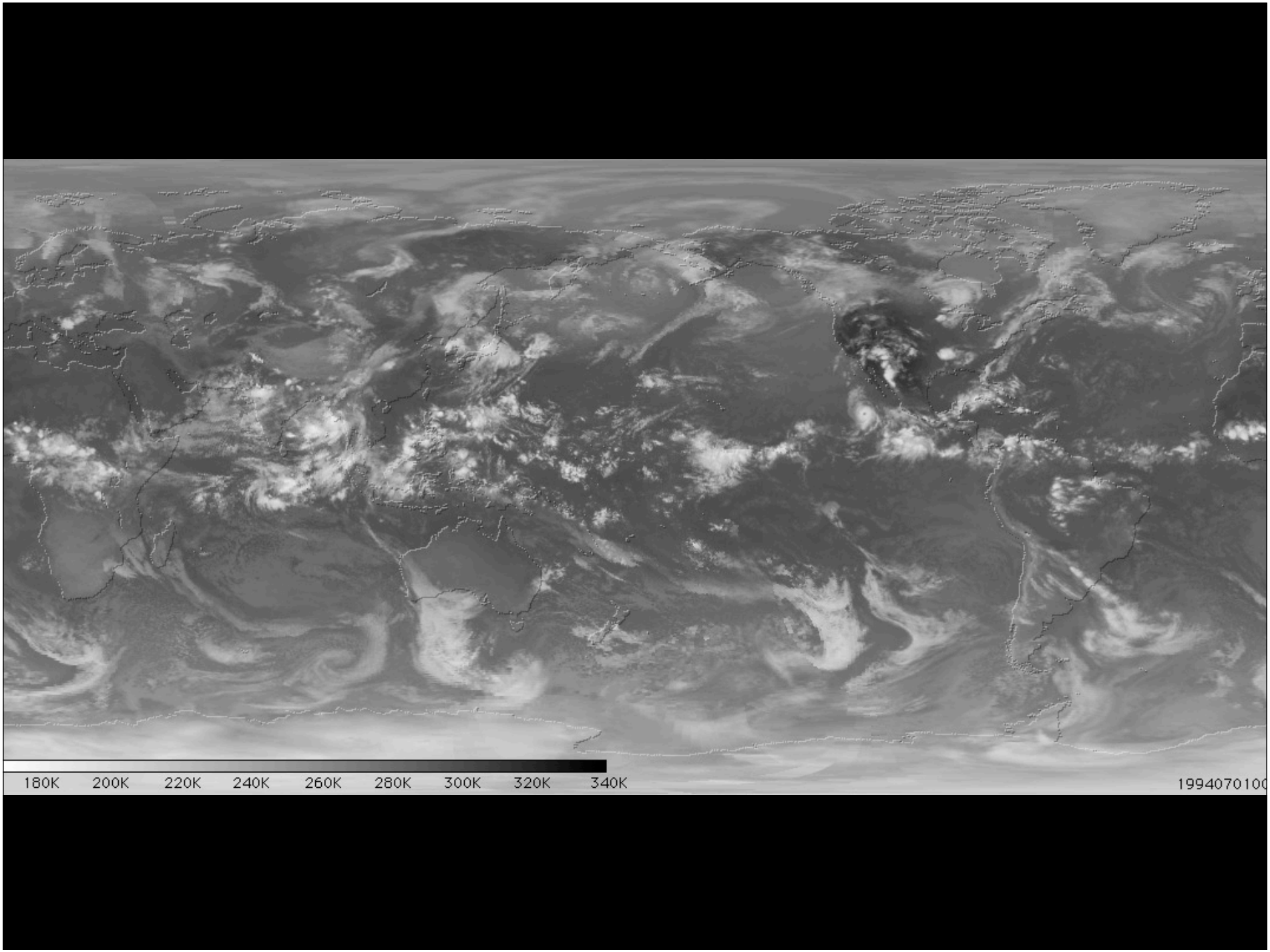
NSF/SIAM CMG workshop
September 16 2011
Washington, DC


180K
200K
220K
240K
260K
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320K
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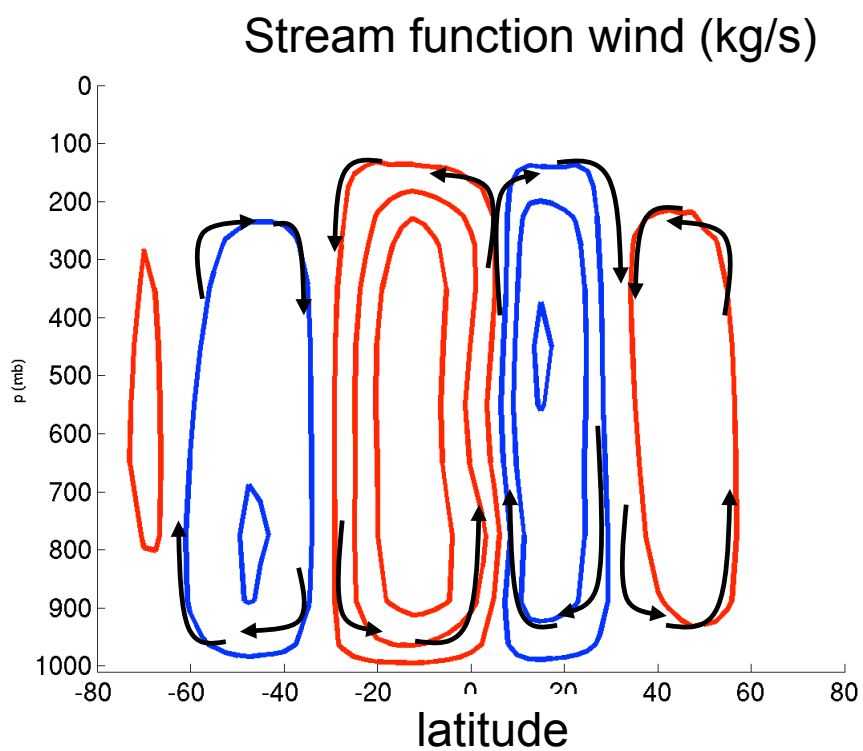
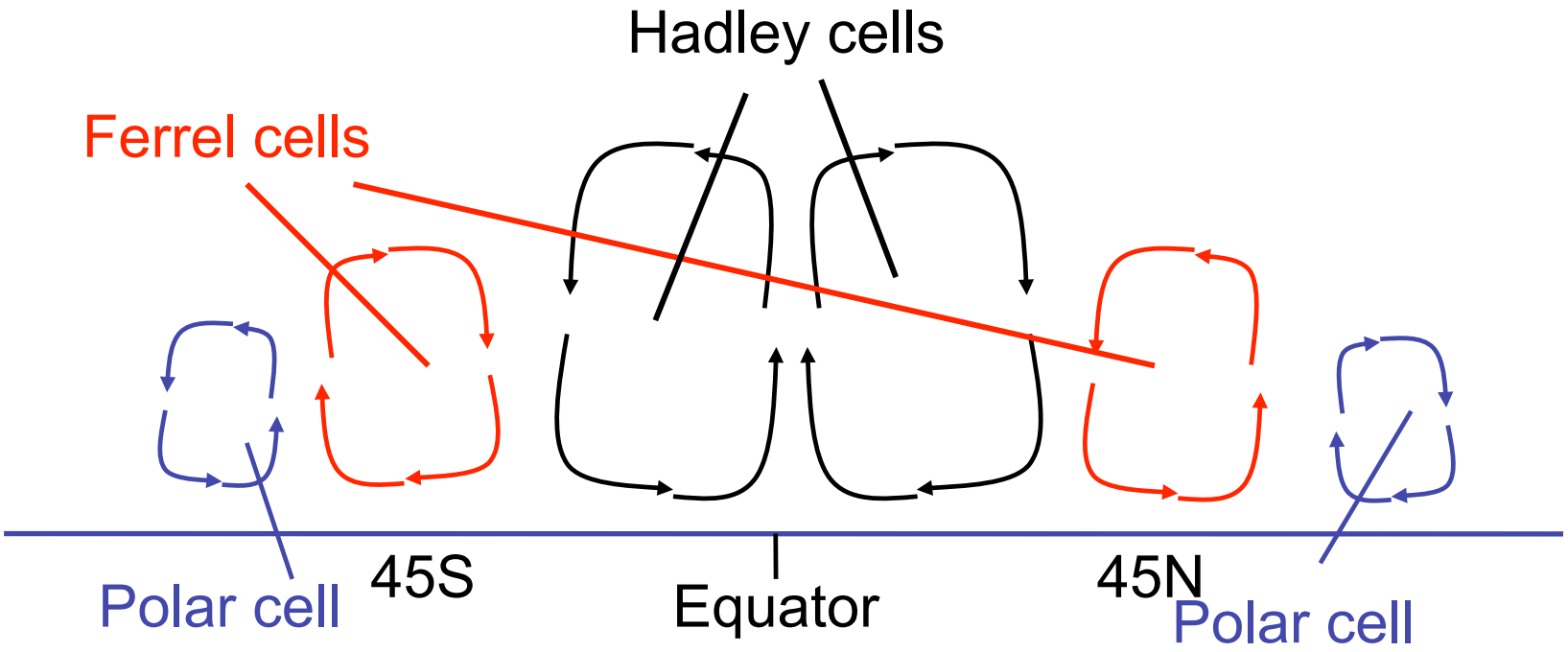
How to describe the circulation?

- This requires some averaging, usual both in time and longitude.
- The circulation can be diagnosed by computing the stream function:

$$\Psi(\varphi,p) = \int_{p}^{p_{surf}} 2\pi \bar{v} a \cos\varphi \frac{dp}{g}$$

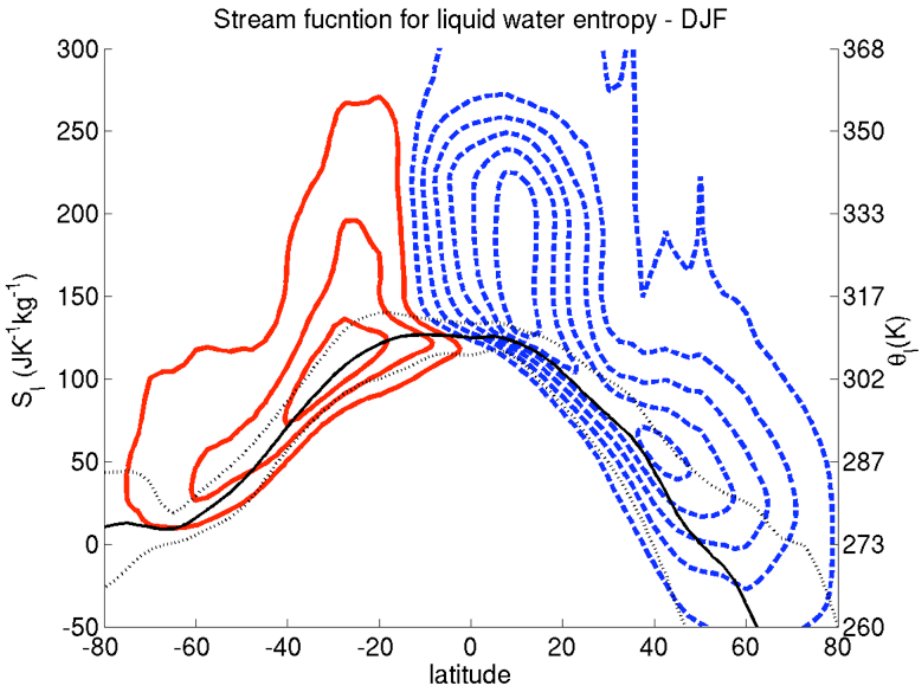




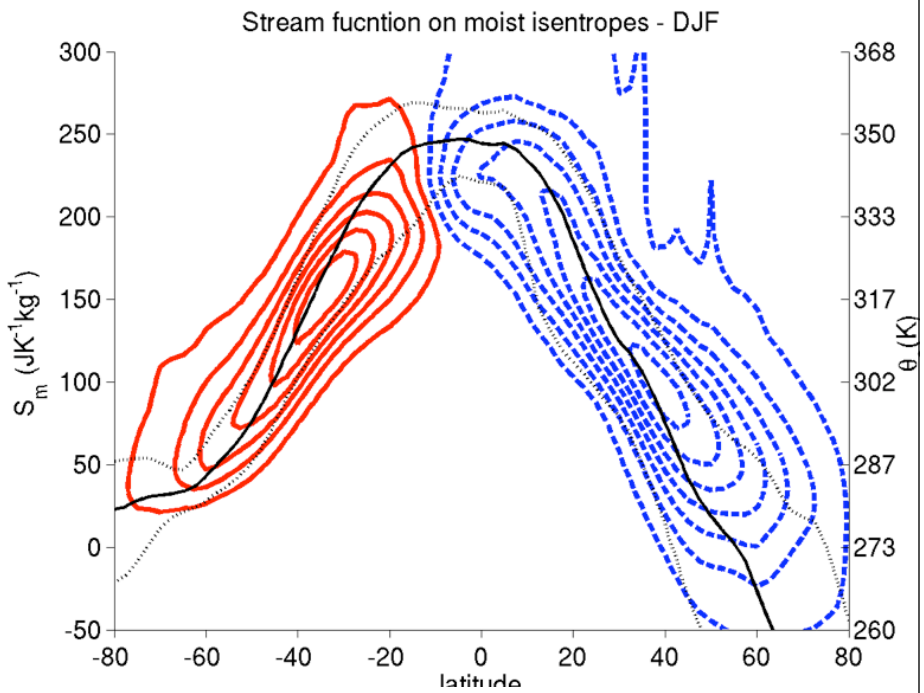
- Eulerian-mean circulation exhibits the ‘classic’ three-cell structure.
- But the Ferrel cell is a reverse circulation that transports energy toward the equator.

- BUT the mean meridional circulation depends very strongly on the vertical coordinate which is used for the averaging.

'dry' circulation averaged on surfaces of constant potential temperature

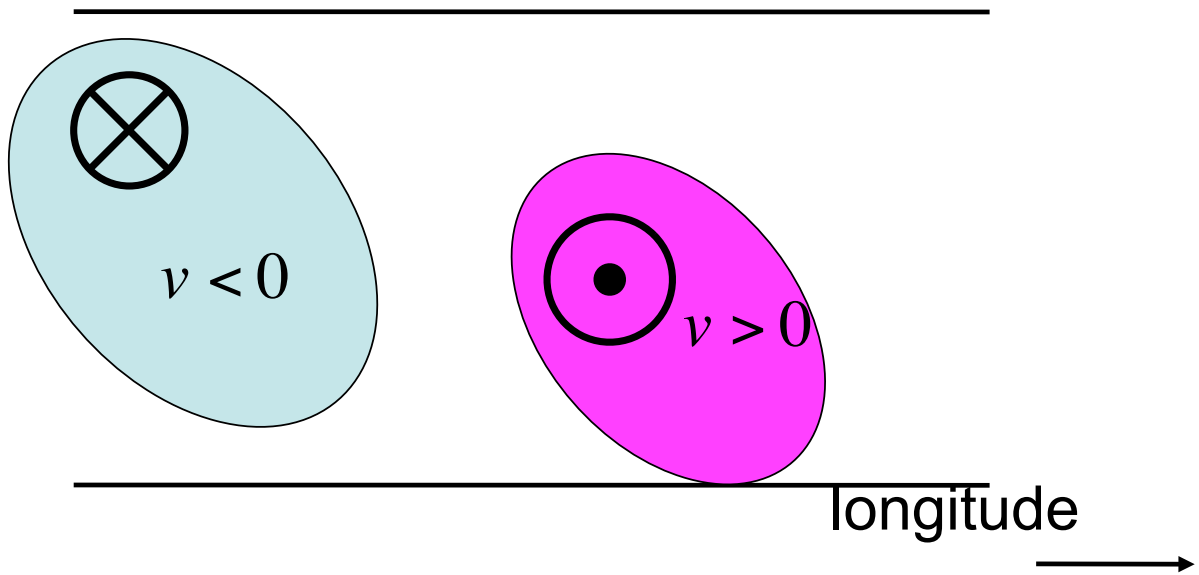


'moist' circulation averaged on surfaces of constant equivalent potential temperature



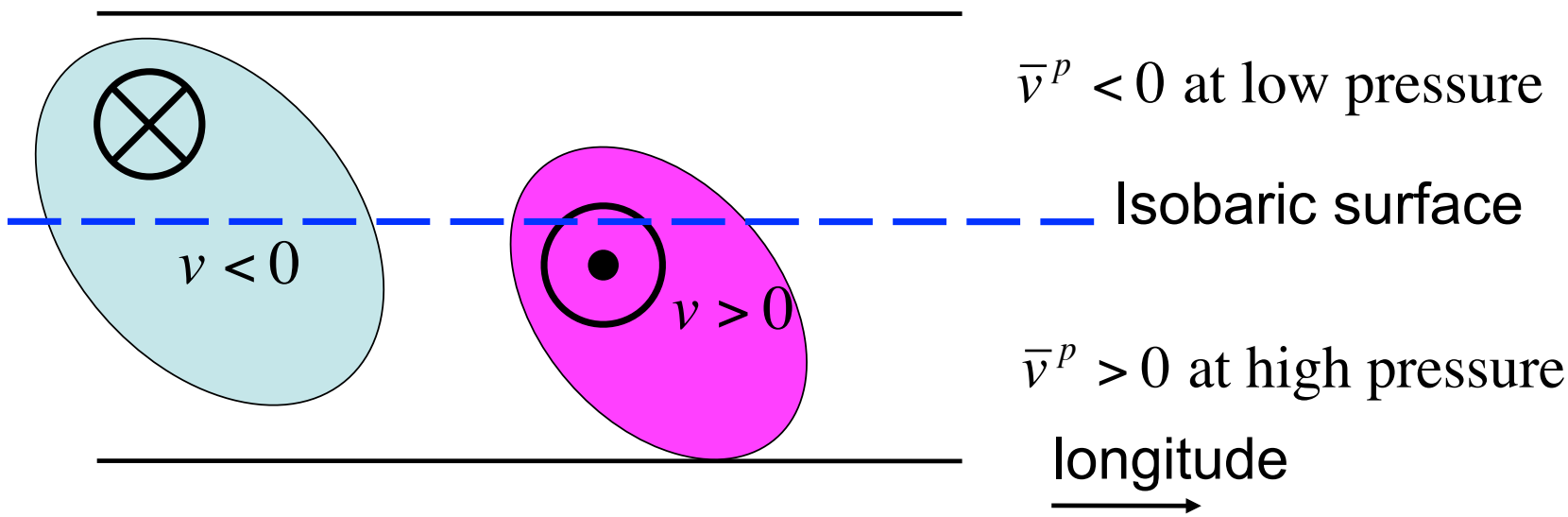
from Pauluis et al.(2010)

Why the circulation in eulerian and isentropic coordinates are in the opposite direction?



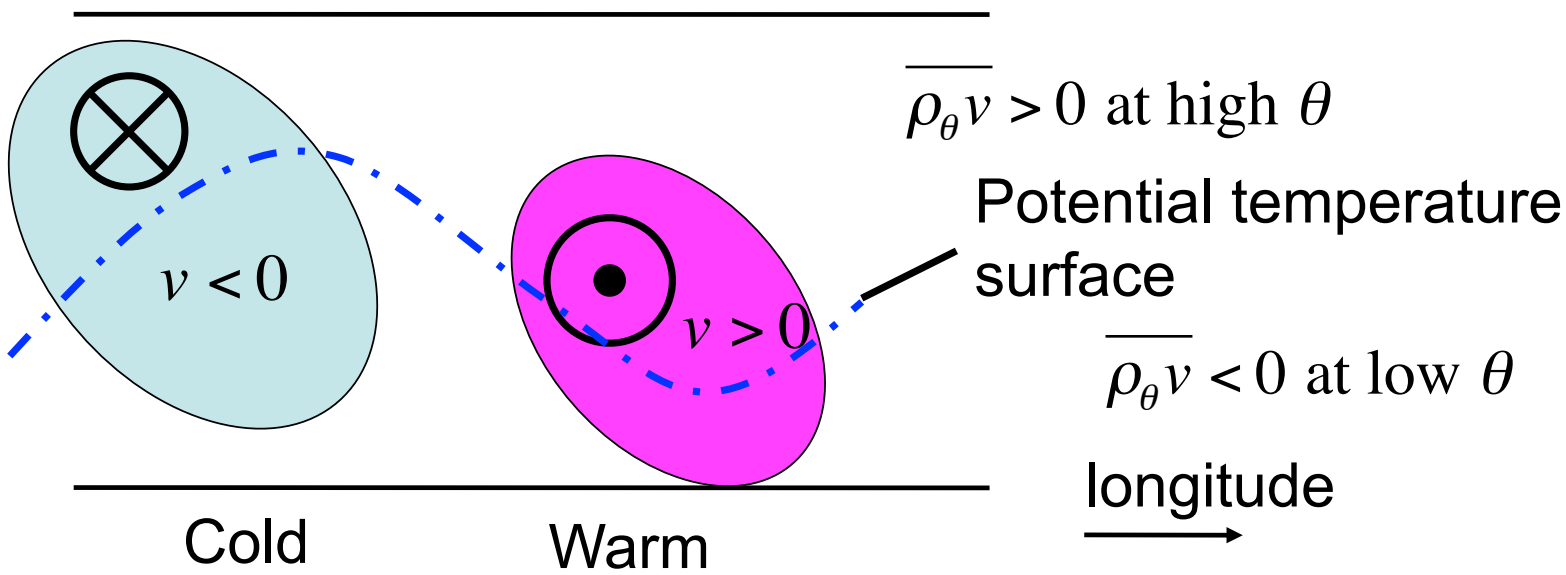
- In the midlatitudes, the flow is highly turbulent: the meridional velocity alternates between poleward and equatorward at all levels.

In the stormtracks: Eulerian-mean circulation



- In the midlatitudes, the flow is highly turbulent: the meridional velocity alternates between poleward and equatorward at all levels.
- This idealized eddies is associated with a poleward flow at high pressure/low level, and equatorward flow at high level

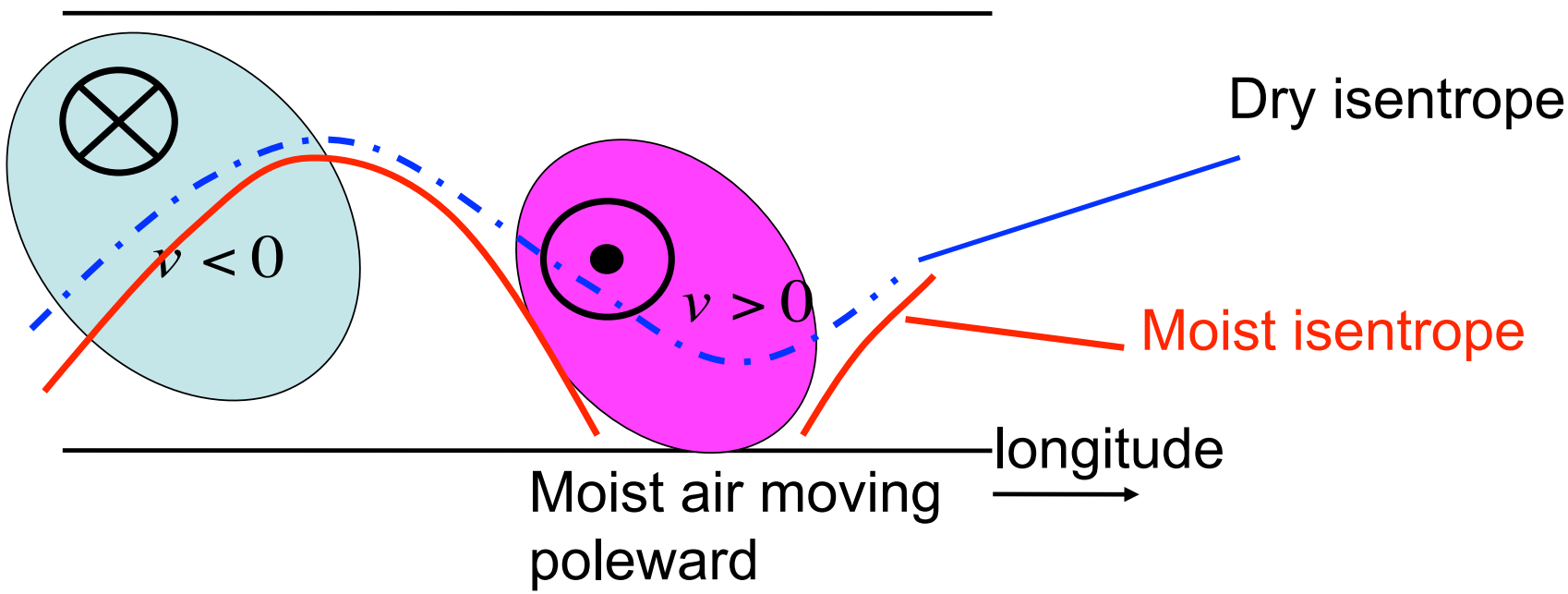
In the stormtracks: Isentropic circulation



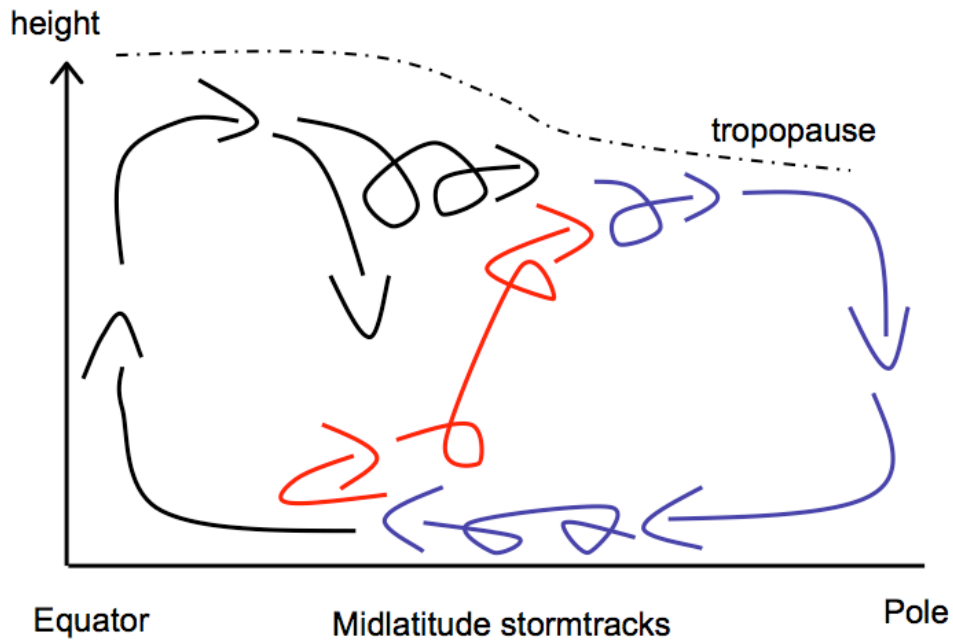
Thickness variations are such that the upper isentropic layer encompass larger fraction of the poleward flow.

Such pattern also corresponds to a net poleward energy mass transport.

In the stormtracks: Circulation on moist isentropes



- Moist isentropes found in the upper troposphere also intersects the Earth's surface.
- Such situation corresponds to a poleward flow of warm, moist air near the surface.



Circulation on dry isentropes

'Moist' branch: additional mass flow on moist isentropes

- The global circulation has two poleward components in the midlatitudes:
  - an upper tropospheric branch of high θ_e-θ_l;
  - an a lower branch of warm, most air with high θ_e-low θ_l, which ascent into the upper troposphere within the stormtracks.
- Mass transport is comparable in each branch.

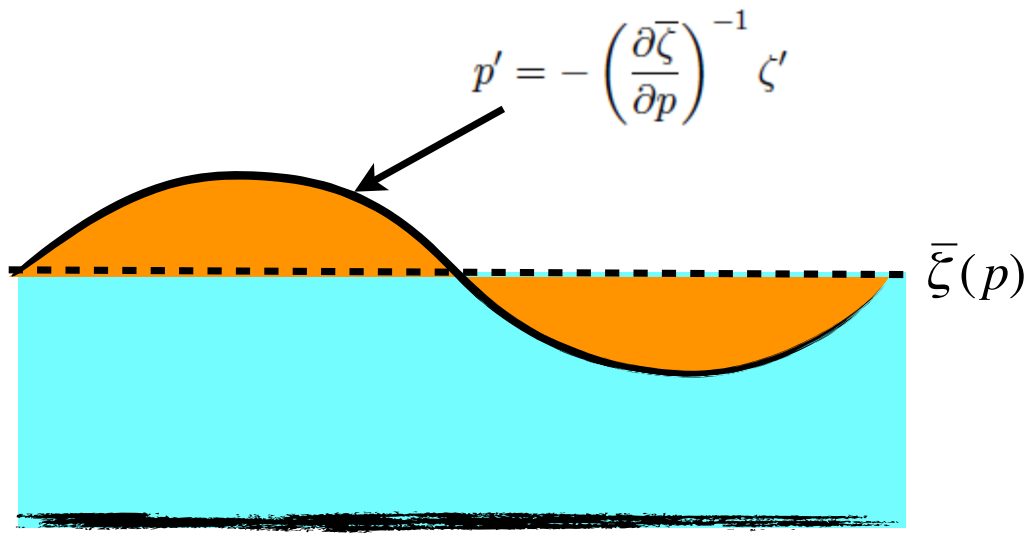
- The streamfunction in an arbitrary coordinate can be defined as

$$\Psi_\zeta(\phi,\zeta_0) = \frac{1}{T}\int_0^T \int_0^{p_{surf}} \int_0^{2\pi} \frac{a\cos\phi}{g} v H(\zeta_0 - \zeta(\lambda,\phi,p,t)) d\lambda\, dp\, dt$$

- It is straightforward to compute given 4 dimensional data.
- How to recover it if we are only given the time mean circulation and eddy statistics?

$$\boxed{\overline{v}, \overline{\zeta}, \overline{v'^2}, \overline{v'\zeta'}, \overline{\zeta'^2}(\phi,p)} \xrightarrow{\;?\;} \boxed{\Phi_\zeta(\phi,\zeta)}$$

Transformed Eulerian Mean (TEM) Circulation

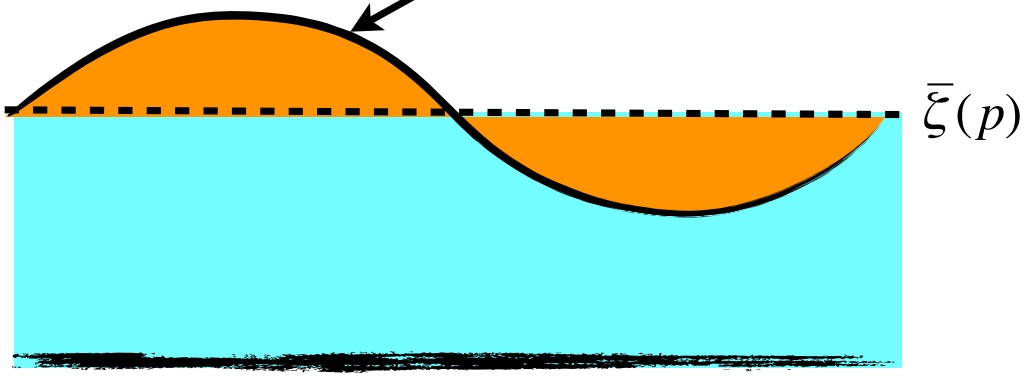


TEM residual circulation:

$$\Psi_\zeta(\phi, \overline{\zeta}(p)) = \underbrace{\int_0^p \overline{v} dp}_{\text{Eulerian-mean circulation}} - \underbrace{\left(\frac{\partial \overline{\zeta}}{\partial p}\right)^{-1} \overline{v'\zeta'}}_{\text{Eddy component}} + \text{h.o.t}$$

Transformed Eulerian Mean (TEM) Circulation

$$p' = -\left(\frac{\partial \overline{\zeta}}{\partial p}\right)^{-1} \zeta'$$



Great, but TEM breaks down when $\overline{\zeta}(p)$ is not stratified

TEM residual circulation:

$$\Psi_\zeta(\phi, \overline{\zeta}(p)) = \underbrace{\int_0^p \overline{v} dp}_{\text{Eulerian-mean circulation}} - \underbrace{\left(\frac{\partial \overline{\zeta}}{\partial p}\right)^{-1} \overline{v'\zeta'}}_{\text{Eddy component}} + \text{h.o.t}$$

The Statistical Transformed Eulerian Mean (STEM) Circulation

We introduce a join distribution of the meridional mass transport $m(\phi, p, \zeta)$ that the streamfunction can be written as

$$\Psi_\zeta(\phi, \zeta) = \int_{-\infty}^{\zeta} \int_{0}^{p_{surf}} m(\phi, p, \tilde{\zeta}) dp \, d\tilde{\zeta}$$

We assume that at each pressure and latitude, the velocity and ζ obey a Gaussian distribution

$$f(v, \zeta) = \frac{1}{2\pi |\Sigma|^{1/2}} \exp\left(-\frac{1}{2} (v', \zeta')^T \Sigma^{-1} (v', \zeta') \right)$$

with the covariance matrix

$$\Sigma = \begin{vmatrix} \overline{v'^2} & \overline{v'\zeta'} \\ \overline{v'\zeta'} & \overline{\zeta'^2} \end{vmatrix}$$

Under these assumptions, the mean velocity for a given value of ζ is

$$E(v|\zeta) = \overline{v} + \frac{\overline{v'\zeta'}}{\overline{\zeta'^2}}(\zeta - \overline{\zeta})$$

The join distribution follows:

$$m(\phi, p, \zeta) = \frac{2\pi a \cos\phi}{g} E(v|\zeta)\mathcal{N}_Z(\overline{\zeta}, \overline{\zeta'^2})$$

$$= m_{mean}(\phi, p, \zeta) + m_{eddy}(\phi, p, \zeta)$$

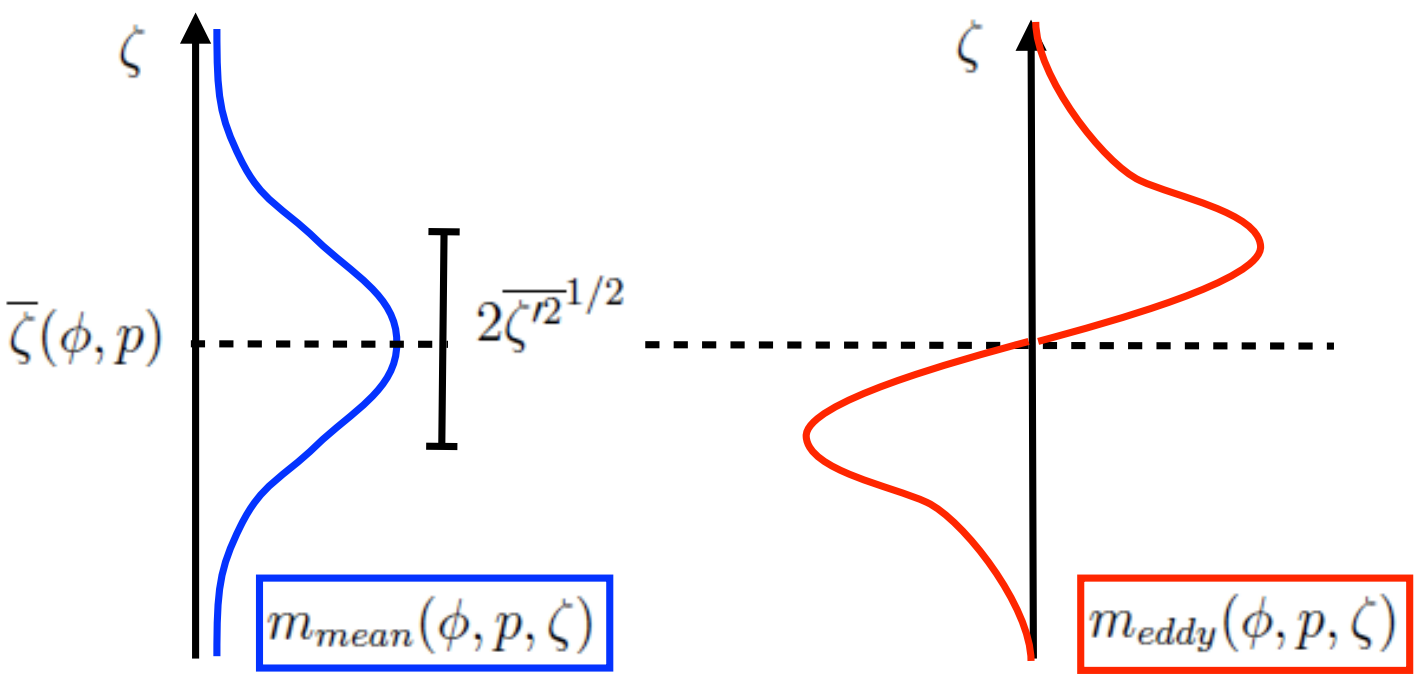
with

$$m_{mean}(\phi, p, \zeta) = \frac{\sqrt{2\pi} a \cos\phi}{g\,\overline{\zeta'^2}^{1/2}} \overline{v} \exp\left(\frac{-(\zeta - \overline{\zeta})^2}{2\overline{\zeta'^2}}\right)$$

$$m_{eddy}(\phi, p, \zeta) = \frac{\sqrt{2\pi} a \cos\phi}{g\,\overline{\zeta'^2}^{3/2}} \overline{v'\zeta'}(\zeta - \overline{\zeta}) \exp\left(\frac{-(\zeta - \overline{\zeta})^2}{2\overline{\zeta'^2}}\right)$$

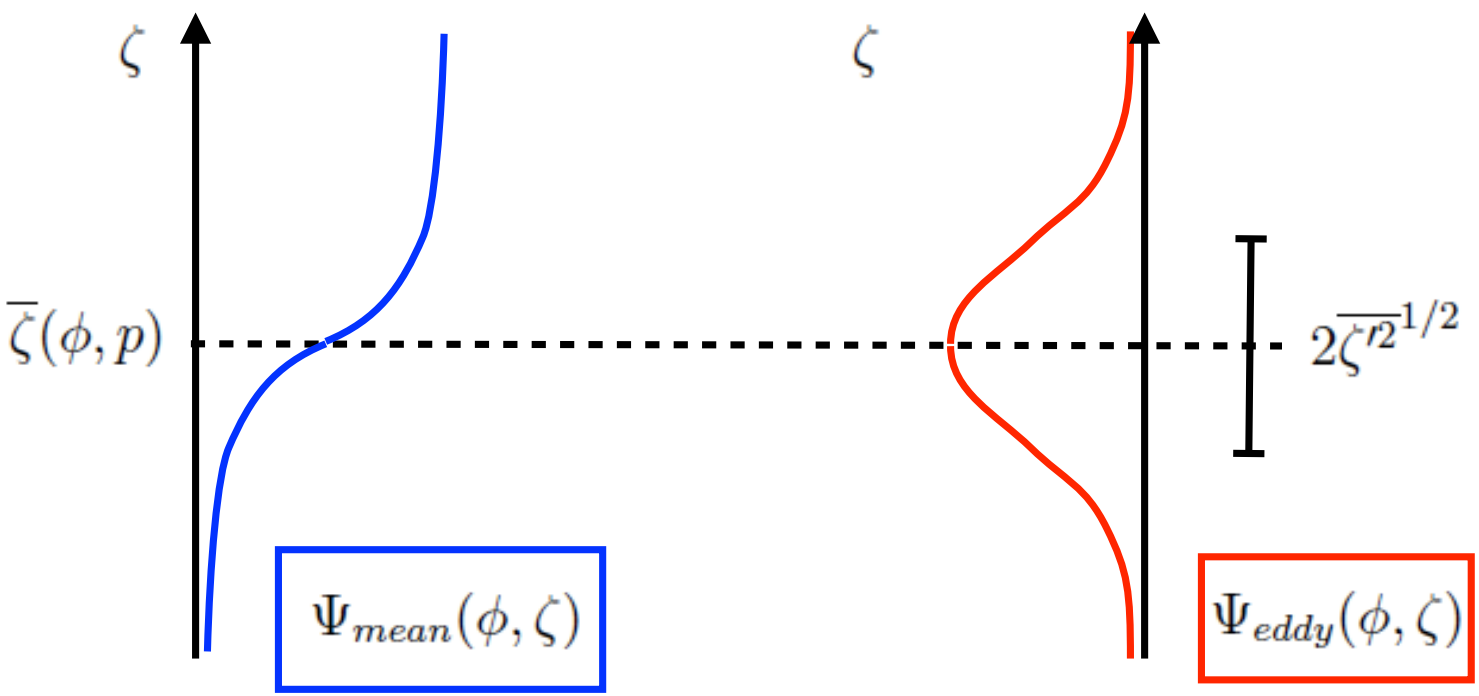
$$m_{mean}(\phi, p, \zeta) = \frac{\sqrt{2\pi} a \cos\phi}{g\,\overline{\zeta'^2}^{1/2}} \overline{v} \exp\left(\frac{-(\zeta - \overline{\zeta})^2}{2\overline{\zeta'^2}}\right)$$

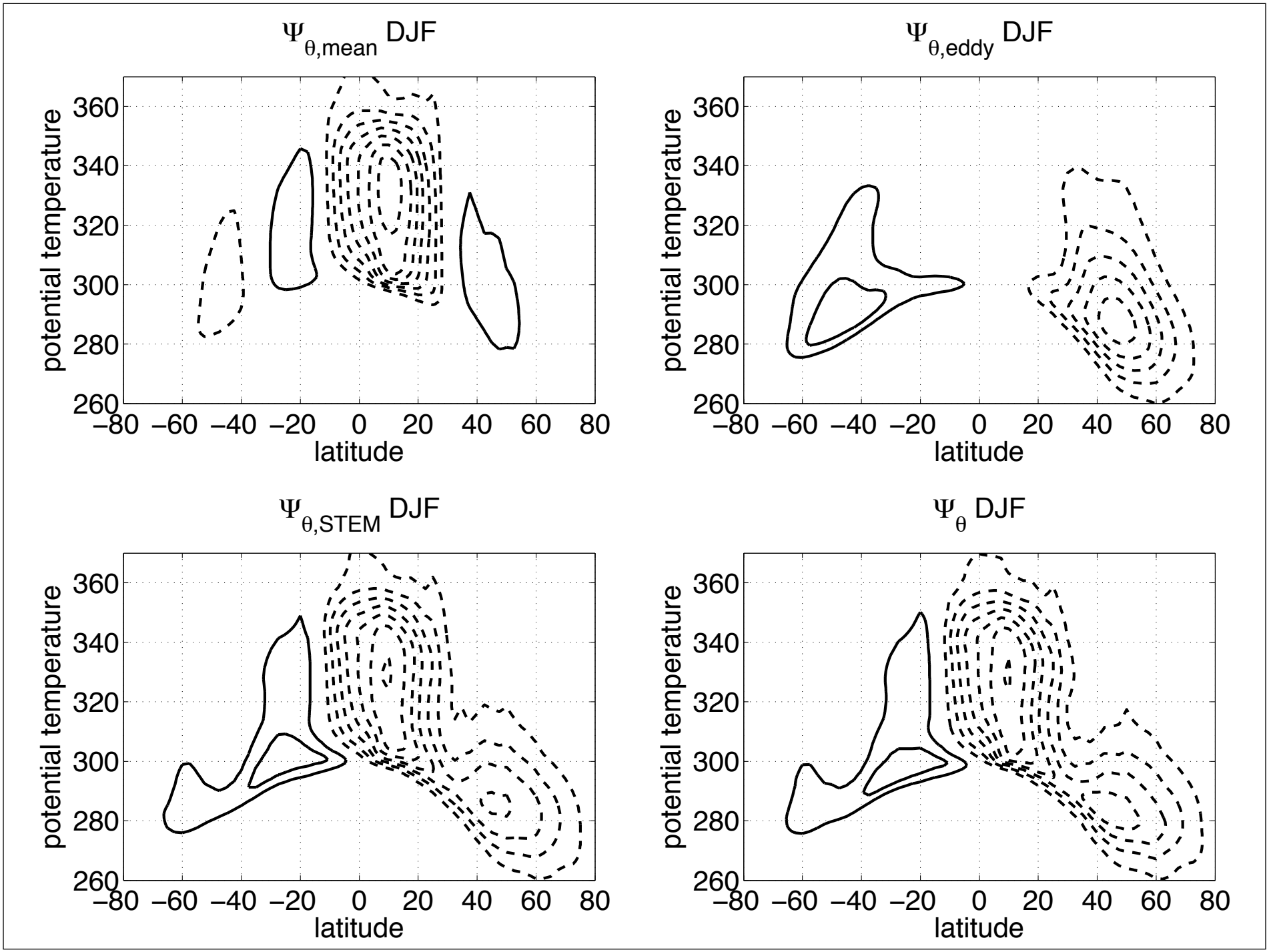
$$m_{eddy}(\phi, p, \zeta) = \frac{\sqrt{2\pi} a \cos\phi}{g\,\overline{\zeta'^2}^{3/2}} \overline{v'\zeta'}(\zeta - \overline{\zeta}) \exp\left(\frac{-(\zeta - \overline{\zeta})^2}{2\overline{\zeta'^2}}\right)$$

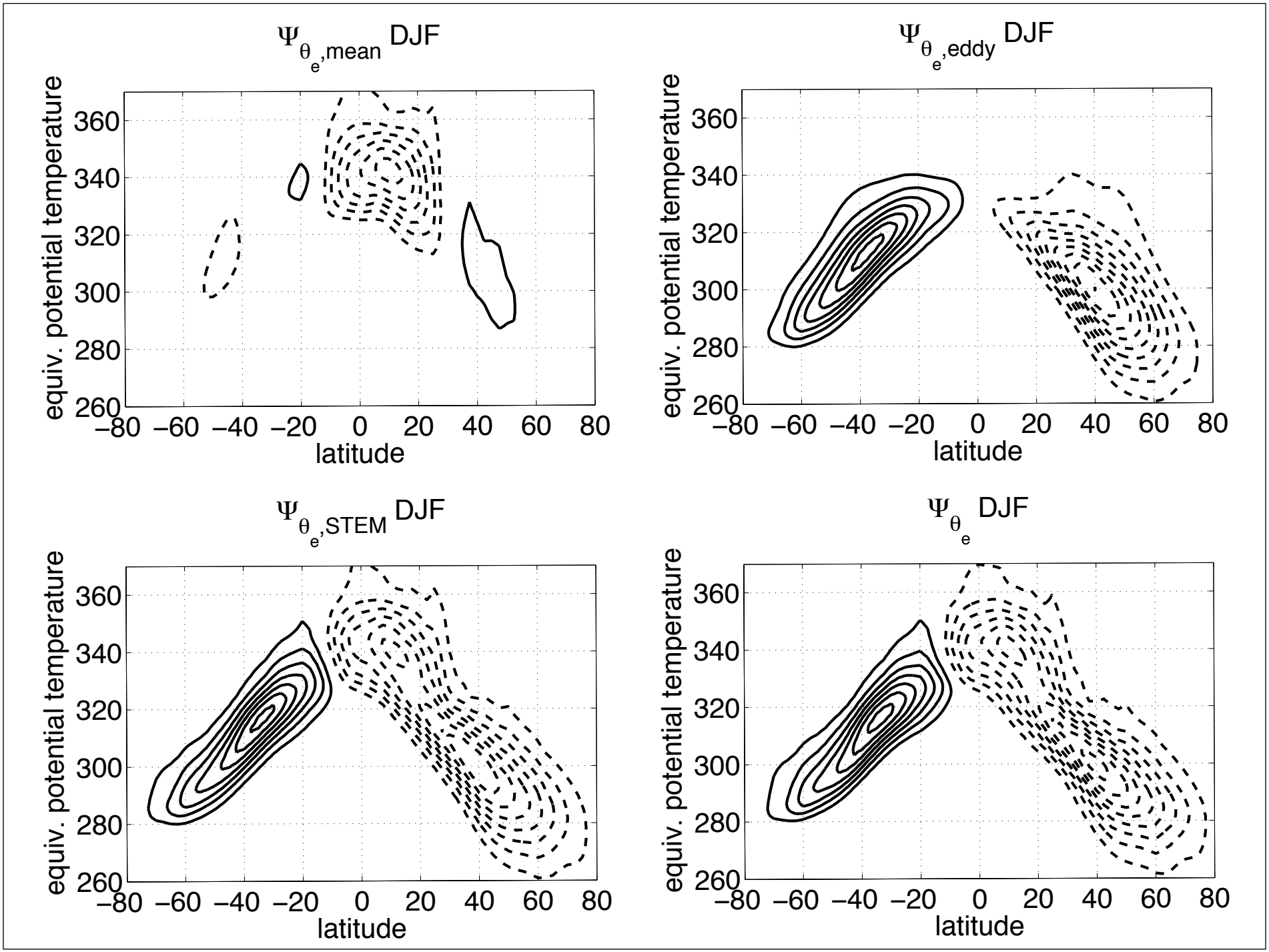


$$\Psi_{mean}(\phi,\zeta) = \frac{\pi a \cos\phi}{g} \int_0^\infty \overline{v} \left(1 + \mathrm{erf}\left(\frac{\zeta - \overline{\zeta}}{\sqrt{2\overline{\zeta'^2}}}\right)\right) dp$$

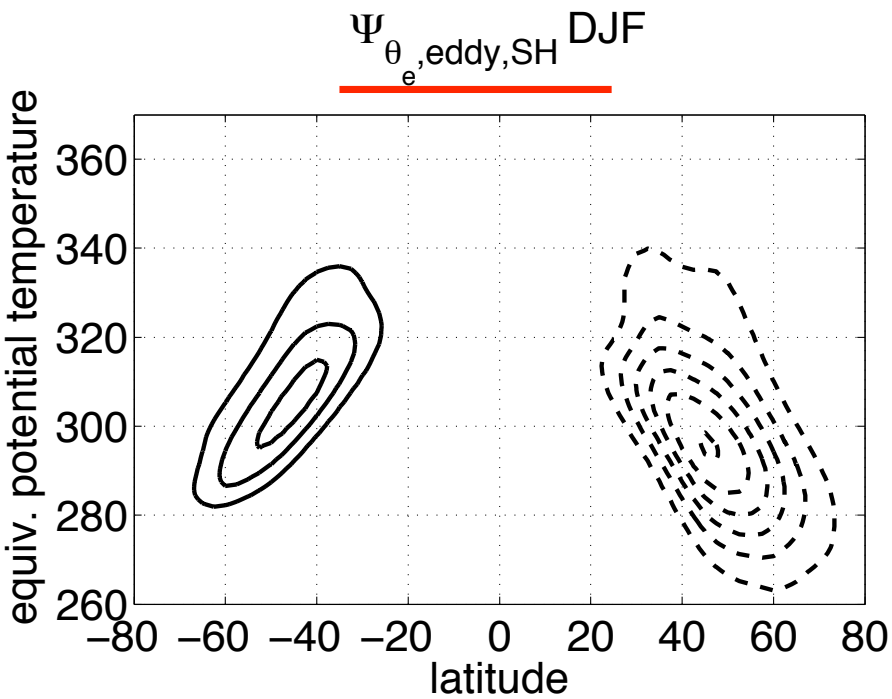
$$\Psi_{eddy}(\phi,\zeta) = \frac{-2\pi a \cos\phi}{g} \int_0^\infty \frac{1}{\sqrt{2\pi\,\overline{\zeta'^2}}} \overline{v'\zeta'} \exp\left(\frac{-(\zeta - \overline{\zeta})^2}{2\overline{\zeta'^2}}\right) dp$$

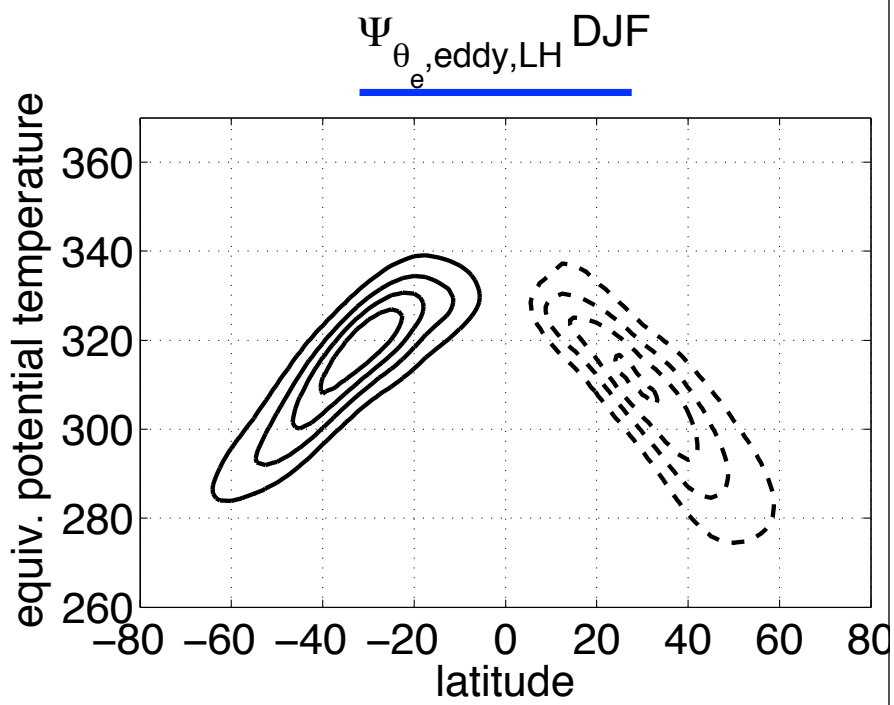


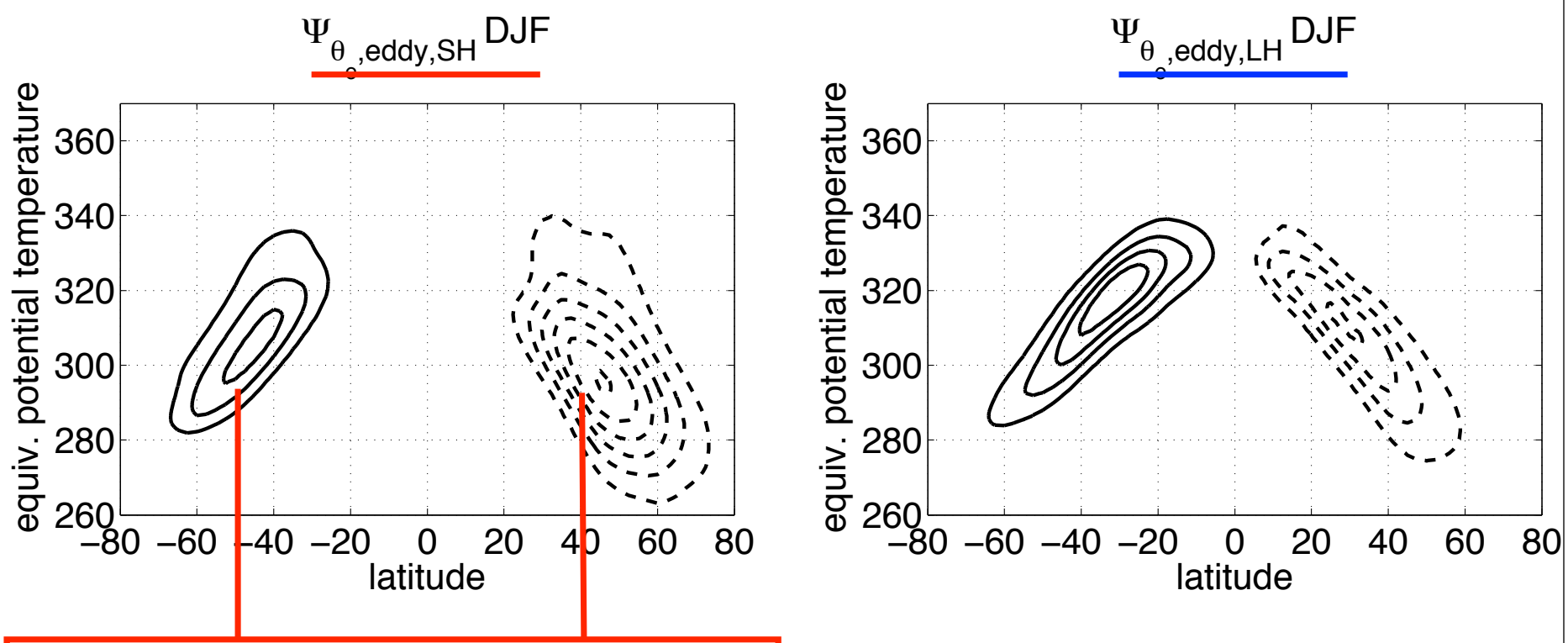

$\Psi_{\theta,mean}$ DJF
$\Psi_{\theta,eddy}$ DJF
$\Psi_{\theta,STEM}$ DJF
Ψ_{θ} DJF
potential temperature
latitude
360
340
320
300
280
260
−80 −60 −40 −20 0 20 40 60 80


$\Psi_{\theta_e,mean}$ DJF
$\Psi_{\theta_e,eddy}$ DJF
$\Psi_{\theta_e,STEM}$ DJF
Ψ_{θ_e} DJF
equiv. potential temperature
latitude
260
280
300
320
340
360
−80
−60
−40
−20
0
20
40
60
80

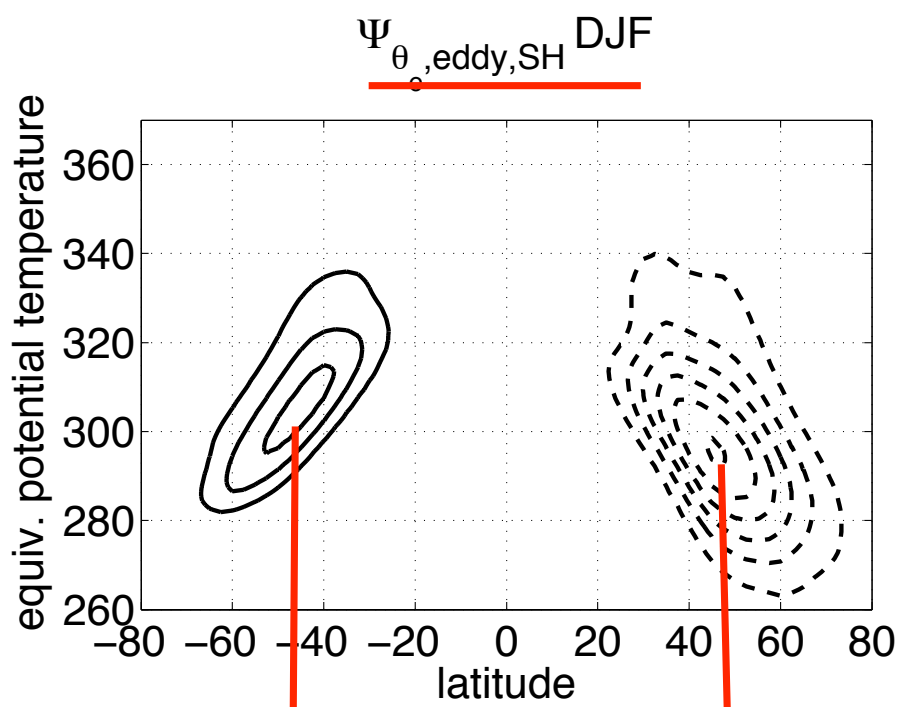
$$\overline{v'\theta'_e} \approx \overline{v'\theta'} + \frac{L}{c_p}\overline{v'q'} \Rightarrow \Psi_{\theta_e,eddy}(\theta_e,\phi) = \Psi_{\theta_e,SH}(\theta_e,\phi) + \Psi_{\theta_e,LH}(\theta_e,\phi)$$

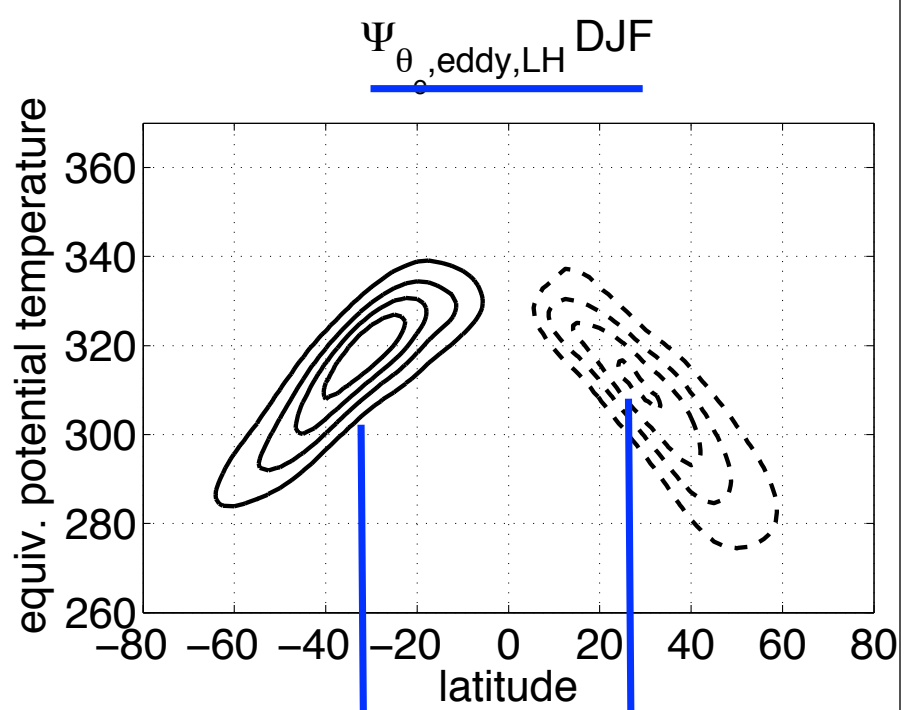






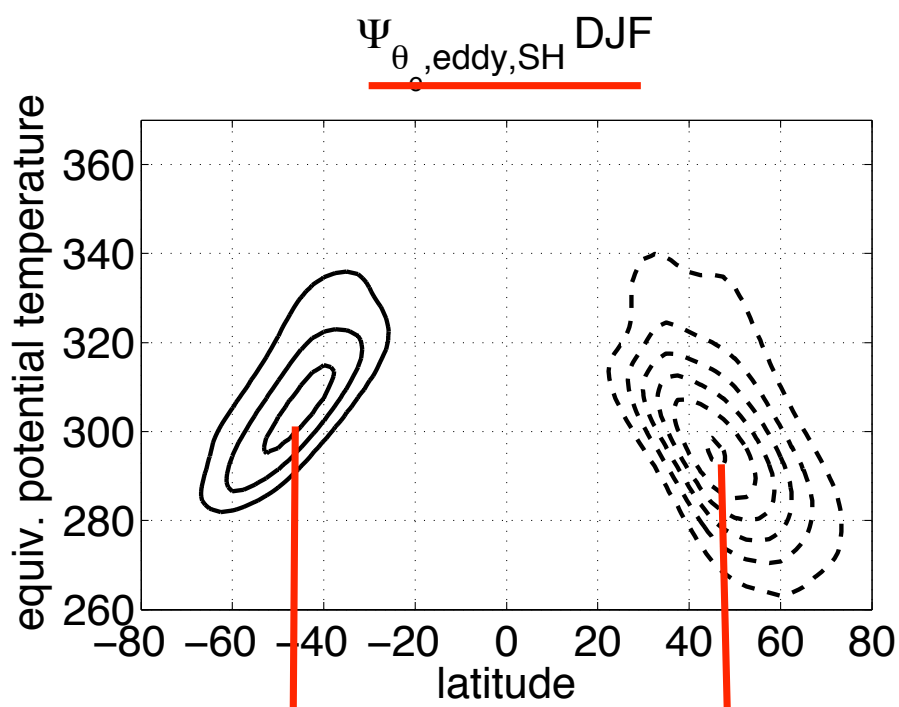
Sensible heat contribution peaks in the midlatitudes

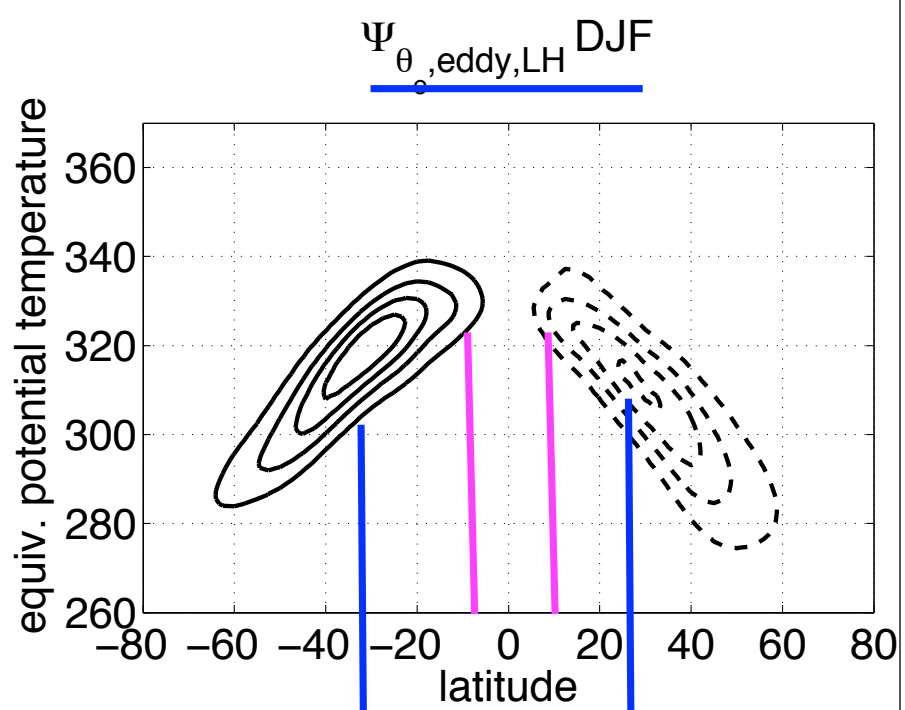




Sensible heat contribution peaks in the midlatitudes

Latent heat contribution peaks on the equatorward side of the stormtracks

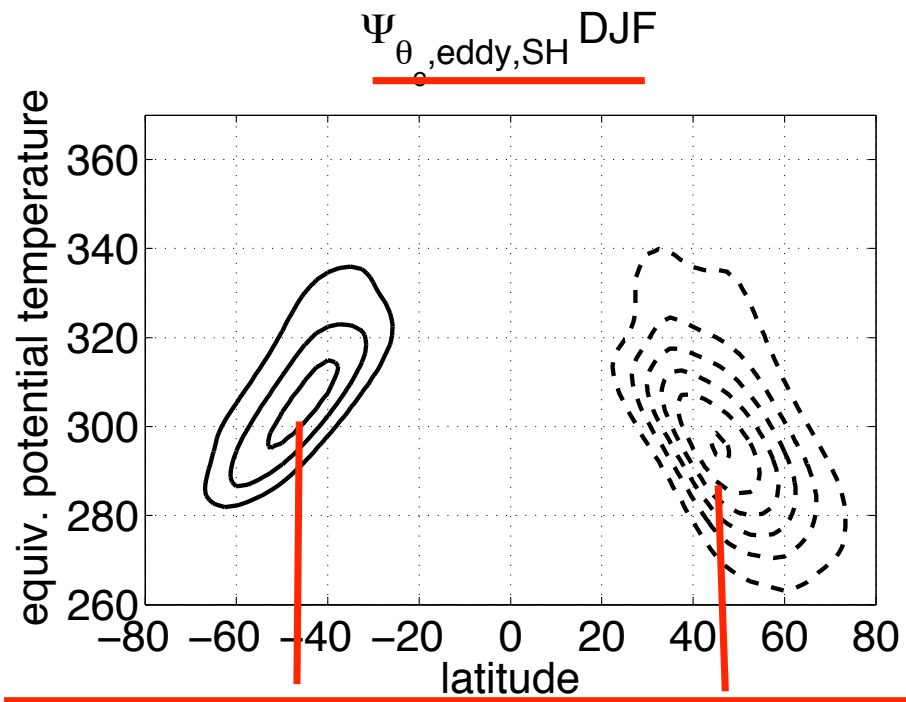


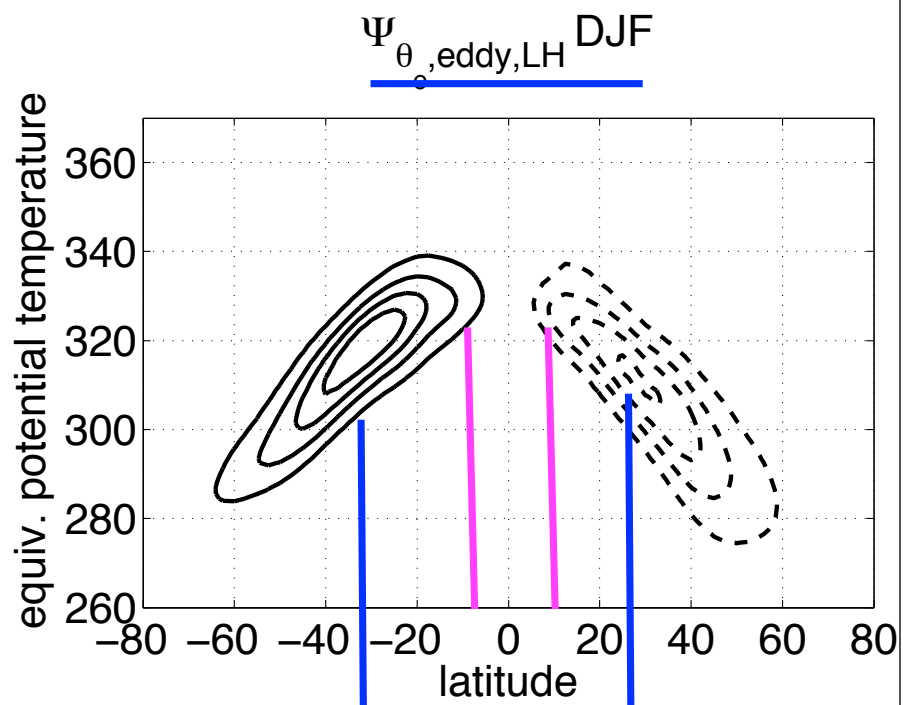


Sensible heat contribution peaks in the midlatitudes

Latent heat contribution peaks on the equatorward side of the stormtracks

Latent heat contribution extends far in the tropics





Sensible heat contribution peaks in the midlatitudes

Latent heat contribution peaks on the equatorward side of the stormtracks

Both contributions overlap across the stormtracks, so that the total circulation is larger than either component.

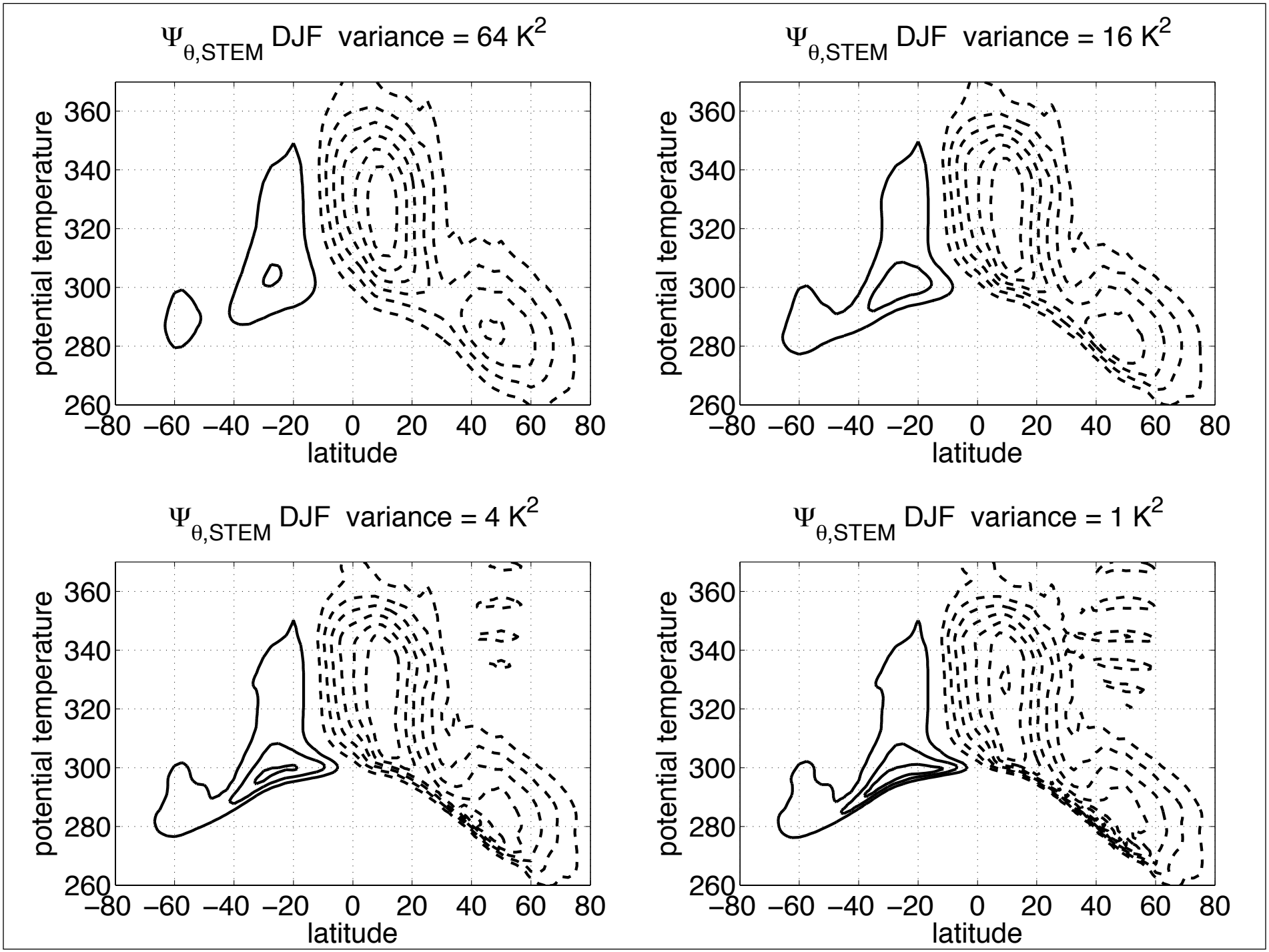
Latent heat contribution extends far in the tropics

Relationship between TEM and STEM

- It can be shown formally that as the variance $\overline{\zeta'^2}$ goes to 0, the STEM streamfunction converges toward the TEM, i.e:

$$\lim_{\overline{\zeta'^2}\to 0} \Psi_{mean}\left(\phi, \overline{\zeta}(p)\right) = \Psi_{TEM,mean}(\phi, p)$$

$$\lim_{\overline{\zeta'^2}\to 0} \Psi_{eddy}(\phi, \overline{\zeta}(p)) = \left(\frac{\partial \overline{\zeta}}{\partial p}\right)^{-1} \overline{v'\zeta'}(\phi, p) = \Psi_{TEM,eddy}(p, \phi)$$


$\Psi_{\theta,STEM}$ DJF variance = 64 K^2
$\Psi_{\theta,STEM}$ DJF variance = 16 K^2
$\Psi_{\theta,STEM}$ DJF variance = 4 K^2
$\Psi_{\theta,STEM}$ DJF variance = 1 K^2
potential temperature
360
340
320
300
280
260
−80 −60 −40 −20 0 20 40 60 80
latitude

Conclusions

- The ‘mean’ atmospheric circulation is highly sensitive to the averaging method.
- The STEM circulation extends the TEM circulation to an arbitrary coordinate system by taking advantage of additional eddy statistics.
- The TEM circulation corresponds to the small variance limit of the STEM circulation.

Some thoughts on CMG:

- The need for new mathematical ideas in geoscience includes the development of new conceptual and theoretical framework.
- Water vapor and clouds remain a central problem in atmospheric/ climate sciences.
- Statistical and stochastic approaches could be potentially very useful in atmospheric and oceanic science (for parameterization, data assimilation and parameter estimation).
- Progress is often serendipitous: it hard to predict which mathematical tool will solve a given physical problems. Active collaborations are keys to sustain successful exchange of ideas.